10.1 Tangents to Circles

Goals
- Identify segments and lines related to circles.
- Use properties of a tangent to a circle.

VOCABULARY

Circle The set of all points in a plane that are equidistant from a given point, called the center of the circle

Radius The distance from the center of a circle to a point on the circle. A segment whose endpoints are the center of the circle and a point on the circle.

Congruent circles Two circles that have the same radius

Diameter The distance across a circle, through its center. A chord that passes through the center of the circle.

Chord A segment whose endpoints are points on the circle

Secant A line that intersects a circle in two points

Tangent A line that intersects a circle in exactly one point

Tangent circles Coplanar circles that intersect in one point

Concentric Coplanar circles that have a common center

Common tangent A line or segment that is tangent to two coplanar circles

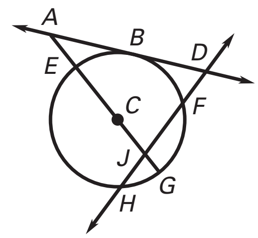
Interior of a circle All points of the plane that are inside a circle

Exterior of a circle All points of the plane that are outside a circle

Point of tangency The point at which a tangent line intersects the circle to which it is tangent

Example 1 *Identifying Special Segments and Lines*

Tell whether the line or segment is best described as a *chord*, a *secant*, a *tangent*, a *diameter*, or a *radius* of $\odot C$.



a. $\overline{CG}$ **b.** $\overline{EG}$ **c.** $\overleftrightarrow{AD}$

Solution

a. $\overline{CG}$ is a radius because C is the center and G is a point on the circle.

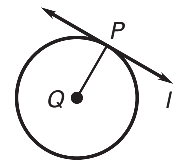
b. $\overline{EG}$ is a diameter because it contains the center C.

c. $\overleftrightarrow{AD}$ is a tangent because it intersects the circle at one point.

THEOREM 10.1

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

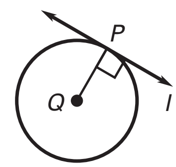
If ℓ is tangent to $\odot Q$ at P, then $\ell \perp \overline{QP}$.



THEOREM 10.2

In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

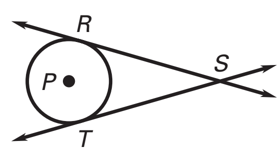
If $\ell \perp \overline{QP}$ at P, then ℓ is tangent to $\odot Q$.



THEOREM 10.3

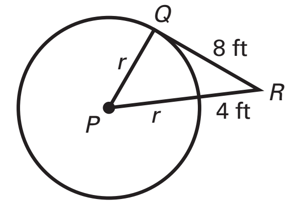
If two segments from the same exterior point are tangent to a circle, then they are congruent.

If $\overleftrightarrow{SR}$ and $\overleftrightarrow{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.



Example 2 *Finding the Radius of a Circle*

You are standing at R, 4 feet from a fountain. The distance from you to a point of tangency on the fountain is 8 feet. What is the radius of the fountain?



Solution

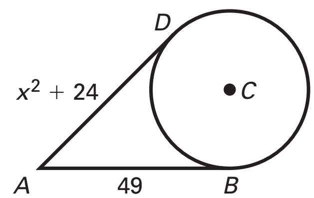
Tangent $\overleftrightarrow{QR}$ is perpendicular to radius $\overline{PQ}$ at Q, so $\triangle PQR$ is a right triangle. So, you can use the Pythagorean Theorem.

$(r + 4)^2 = r^2 + 8^2$	**Pythagorean Theorem**
$r^2 + 8r + 16 = r^2 + 64$	**Square of binomial**
$8r = 48$	**Subtract r^2 and 16 from each side.**
$r = 6$	**Divide.**

Answer The radius of the fountain is 6 feet.

Example 3 *Using Properties of Tangents*

$\overleftrightarrow{AB}$ is tangent to $\odot C$ at B. $\overleftrightarrow{AD}$ is tangent to $\odot C$ at D. Find the value of x.

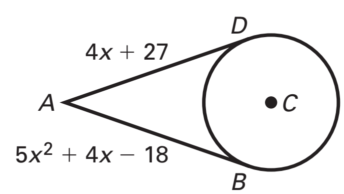


Solution

$AB = AD$	**Use Theorem 10.3.**
$49 = x^2 + 24$	**Substitute.**
$25 = x^2$	**Subtract 24 from each side.**
$\pm 5 = x$	**Find the square roots of 25.**

Checkpoint Complete the following exercise.

1. $\overleftrightarrow{AB}$ is tangent to $\odot C$ at B. $\overleftrightarrow{AD}$ is tangent to $\odot C$ at D. Find the value of x.



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