10.3 Inscribed Angles

Goals
- Use inscribed angles to solve problems.
- Use properties of inscribed polygons.

VOCABULARY

Inscribed angle An angle whose vertex is on a circle and whose sides contain chords of the circle

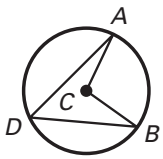
Intercepted arc The arc that lies in the interior of an inscribed angle and has endpoints on the angle

Inscribed polygon A polygon whose vertices all lie on a circle

Circumscribed circle A circle with an inscribed polygon

THEOREM 10.8: MEASURE OF AN INSCRIBED ANGLE

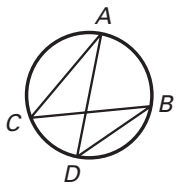
If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.



$m\angle ADB = \frac{1}{2}\underline{m\overset{\frown}{AB}}$

THEOREM 10.9

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

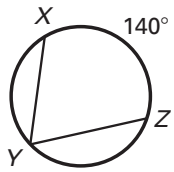


$\angle C \cong \angle \underline{D}$

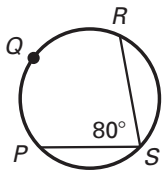
Example 1 *Measures of Arcs and Inscribed Angles*

Find the measure of the arc or angle.

a.



b.



a. $m\angle XYZ = \frac{1}{2}\underline{m\overset{\frown}{XZ}} = \frac{1}{2}(\underline{140}^\circ) = \underline{70}^\circ$

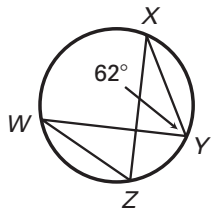
b. $m\overset{\frown}{PQR} = 2m\angle \underline{PSR} = 2(\underline{80}^\circ) = \underline{160}^\circ$

Example 2 *Finding the Measure of an Angle*

It is given that $m\angle Y = 62°$. What is $m\angle Z$?

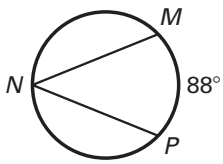
$\angle Y$ and $\angle Z$ both intercept $\overset{\frown}{WX}$, so $\underline{\angle Y} \cong \underline{\angle Z}$.

Answer So, $m\angle \underline{Z} = m\angle \underline{Y} = \underline{62}^\circ$.



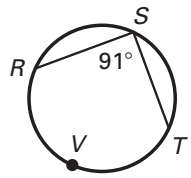
✔ ***Checkpoint*** **Find the measure of the arc or angle.**

1. $\angle MNP$



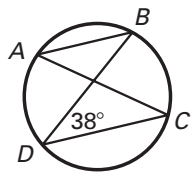
44°

2. $\overset{\frown}{RVT}$



182°

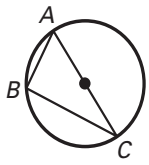
3. $\angle A$



38°

THEOREM 10.10

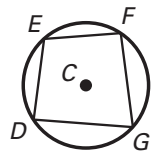
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$\angle$ B is a right angle if and only if $\overline{AC}$ is a diameter of the circle.

THEOREM 10.11

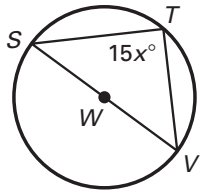
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



D, *E*, *F*, and *G* lie on some circle, $\odot C$, if and only if $m\angle D + m\angle F =$ 180 ° and $m\angle E + m\angle G =$ 180 °.

Example 3 ***Using Theorem 10.10***

Find the value of *x*.



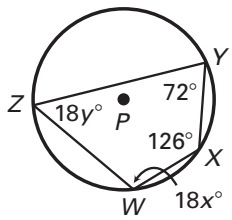
$\overline{SV}$ is a diameter. So, $\angle T$ is a right angle and $m\angle T =$ 90 °.

$15x° =$ 90 °

$x =$ 6

Checkpoint **Complete the following exercise.**

4. In the diagram, *WXYZ* is inscribed in $\odot P$. Find the values of *x* and *y*.



$x = 6$; $y = 3$