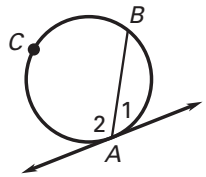
# 10.4 Other Angle Relationships in Circles

**Goals**
- Use angles formed by tangents and chords to solve problems in geometry.
- Use angles formed by lines that intersect a circle to solve problems.

**THEOREM 10.12**

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.




$m\angle 1 = \frac{1}{2}$ $\overset{\frown}{mAB}$, $m\angle 2 = \frac{1}{2}$ $\overset{\frown}{mBCA}$

**Example 1** *Finding an Angle Measure*

**In the diagram below, $\overleftrightarrow{KL}$ is tangent to the circle. Find $m\angle KLM$.**

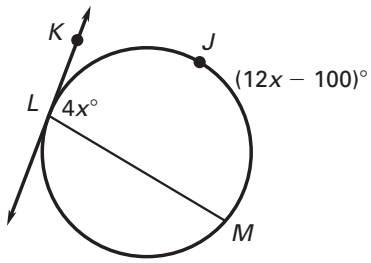
$m\angle KLM = \frac{1}{2}\ m\overset{\frown}{MJL}$

$4x = \frac{1}{2}(12x - 100)$

$8x = 12x - 100$

$100 = 4x$

$25 = x$

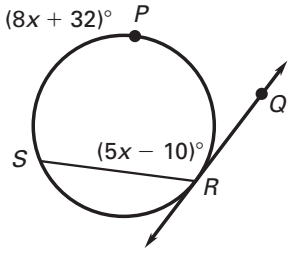



**Answer** $m\angle KLM = (4 \cdot 25)° = 100°$

✔ ***Checkpoint*** **Complete the following exercise.**

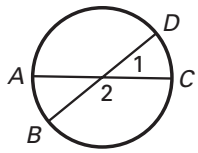
**1.** $\overleftrightarrow{QR}$ is tangent to the circle. Find $m\angle QRS$.

120°



**THEOREM 10.13**

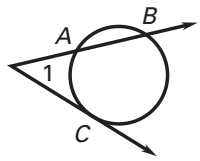
If two chords intersect in the *interior* of a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.



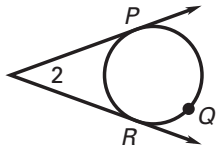

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$, $m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$

**THEOREM 10.14**

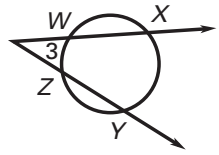
If a tangent and a secant, two tangents, or two secants intersect in the *exterior* of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.




$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$




$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$

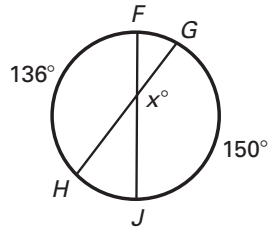



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

**Example 2** *Measure of an Angle Formed by Two Chords*

Find the value of $x$.

**Solution**




$x^\circ = \frac{1}{2}(m\overset{\frown}{FH} + m\overset{\frown}{GJ})$ **Apply Theorem 10.13.**

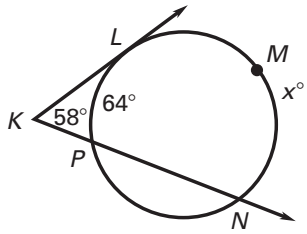
$x^\circ = \frac{1}{2}(136^\circ + 150^\circ)$ **Substitute.**

$x = 143$ **Simplify.**

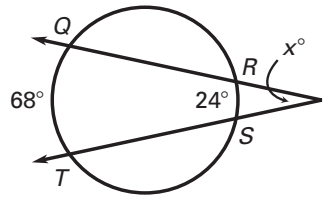
**Example 3** *Using Theorem 10.14*

**Find the value of $x$.**

**a.**




**b.**




**Solution**

**a.** $m\angle LKP = \frac{1}{2}(m\overset{\frown}{LMN} - m\overset{\frown}{LP})$ **Apply Theorem 10.14.**

$\underline{58}^\circ = \frac{1}{2}(x^\circ - \underline{64}^\circ)$ **Substitute.**

$\underline{116} = x - \underline{64}$ **Multiply each side by 2.**

$\underline{180} = x$ **Solve for $x$.**

**b.** $x = \frac{1}{2}(m\overset{\frown}{QT} - m\overset{\frown}{RS})$ **Apply Theorem 10.14.**

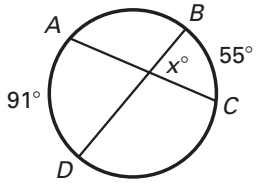
$= \frac{1}{2}(\underline{68} - \underline{24})$ **Substitute.**

$= \frac{1}{2}(\underline{44})$ **Subtract.**

$= \underline{22}$ **Multiply.**

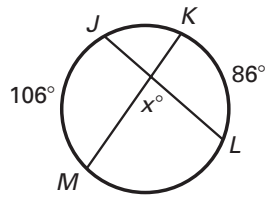
✔ ***Checkpoint*** **Find the value of $x$.**

**2.**



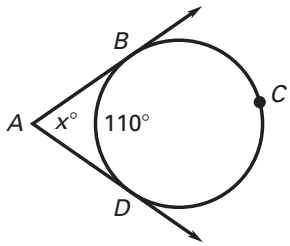

73°

**3.**



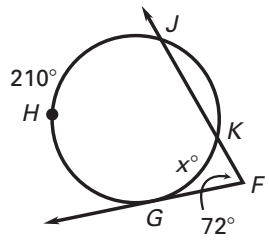

84°

**4.**




70°

**5.**




66°