

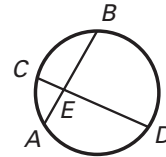
# 10.5

## Segment Lengths in Circles

- Goals**
- Find the lengths of segments of chords.
  - Find the lengths of segments of tangents and secants.

### THEOREM 10.15

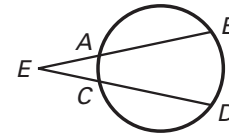
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$$EA \cdot EB = EC \cdot ED$$

### THEOREM 10.16

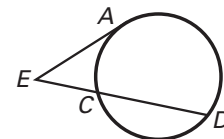
If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.



$$EA \cdot EB = EC \cdot ED$$

### THEOREM 10.17

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.



$$(EA)^2 = EC \cdot ED$$

**Example 1** Finding Segment Lengths

Chords  $\overline{JN}$  and  $\overline{KM}$  intersect inside the circle.  
Find the value of  $x$ .

$$LJ \cdot LN = LK \cdot LM$$

Theorem 10.15

$$10 \cdot x = 8 \cdot 5$$

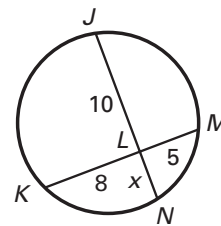
Substitute.

$$10x = 40$$

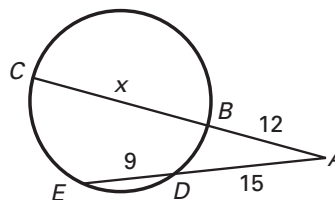
Simplify.

$$x = 4$$

Divide each side by 10.

**Example 2** Finding Segment Lengths

Find the value of  $x$ .

**Solution**

$$AB \cdot AC = AD \cdot AE$$

Theorem 10.16

$$12 \cdot (x + 12) = 15 \cdot (9 + 15)$$

Substitute.

$$12x + 144 = 360$$

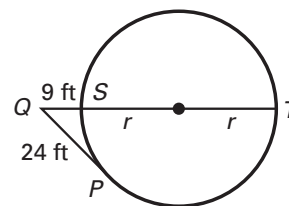
Simplify.

$$x = 18$$

Solve for  $x$ .

**Example 3** Estimating the Radius of a Circle

You are standing at a point  $Q$ , about 9 feet from a large circular tent. The distance from you to a point of tangency on the tent is about 24 feet. Estimate the radius of the tent.

**Solution**

$$(QP)^2 = QS \cdot QT$$

Use Theorem 10.17.

$$24^2 \approx 9 \cdot (2r + 9)$$

Substitute.

$$576 \approx 18r + 81$$

Simplify.

$$495 \approx 18r$$

Subtract 81 from each side.

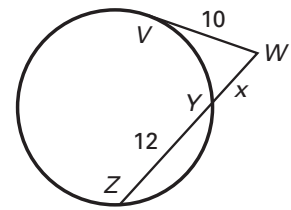
$$27.5 \approx r$$

Divide each side by 18.

**Answer** So, the radius of the tent is about 27.5 feet.

**Example 4** Finding Segment Lengths

Use the figure at the right to find the value of  $x$ .



$$\begin{aligned} (\underline{VW})^2 &= \underline{WY} \cdot \underline{WZ} \\ \underline{10}^2 &= x \cdot (\underline{12} + \underline{x}) \\ 0 &= \underline{x^2} + \underline{12x} - \underline{100} \end{aligned}$$

$$x = \frac{-\underline{12} \pm \sqrt{\underline{12}^2 - 4(\underline{1})(\underline{-100})}}{2}$$

$$x = \underline{-6} \pm \underline{2\sqrt{34}}$$

Use the positive solution, because lengths cannot be negative.

Answer So,  $x = \underline{-6} + \underline{2\sqrt{34}} \approx \underline{5.66}$ .

**Theorem 10.17**

Substitute.

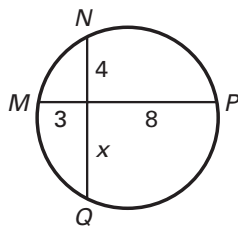
Write in standard form.

Use Quadratic Formula.

Simplify.

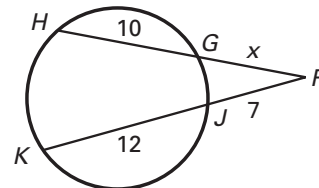
**✓ Checkpoint** Complete the following exercises.

1. Chords  $\overline{MP}$  and  $\overline{NQ}$  intersect inside the circle. Find the value of  $x$ .



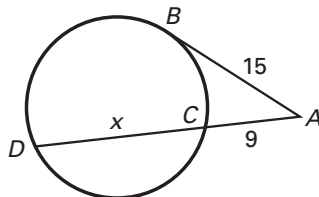
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2. Find the value of  $x$ .



7.57

3. Find the value of  $x$ .



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