

# 10.6

## Equations of Circles

- Goals**
- Write the equation of a circle.
  - Use the equation of a circle and its graph to solve problems.

### VOCABULARY

Standard equation of a circle: A circle with radius  $r$  and center  $(h, k)$  has this standard equation:  $(x - h)^2 + (y - k)^2 = r^2$ .

### Example 1 Writing a Standard Equation of a Circle

Write the standard equation of the circle with center  $(0, 6)$  and radius  $3.6$ .

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2 && \text{Standard equation of a circle} \\(x - \underline{0})^2 + (y - \underline{6})^2 &= \underline{3.6}^2 && \text{Substitute.} \\x^2 + (y - \underline{6})^2 &= \underline{12.96} && \text{Simplify.}\end{aligned}$$

### Example 2 Writing a Standard Equation of a Circle

The point  $(-1, 1)$  is on a circle whose center is  $(-3, 4)$ . Write the standard equation of the circle.

Find the radius. The radius is the distance from the point  $(-1, 1)$  to the center  $(-3, 4)$ .

$$\begin{aligned}r &= \sqrt{(-3 - (-1))^2 + (4 - 1)^2} && \text{Use the Distance Formula.} \\r &= \sqrt{(-2)^2 + 3^2} && \text{Simplify.} \\r &= \sqrt{13} && \text{Simplify.}\end{aligned}$$

Substitute  $(h, k) = (-3, 4)$  and  $r = \sqrt{13}$  into the standard equation of a circle.

$$\begin{aligned}(x - (-3))^2 + (y - 4)^2 &= (\sqrt{13})^2 && \text{Standard equation of a circle} \\(x + 3)^2 + (y - 4)^2 &= 13 && \text{Simplify.}\end{aligned}$$

**Example 3** Graphing a Circle

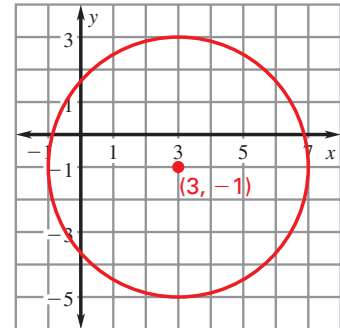
The equation of a circle is  $(x - 3)^2 + (y + 1)^2 = 16$ . Graph the circle.

Rewrite the equation to find the center and radius:

$$(x - 3)^2 + (y + 1)^2 = 16$$

$$(x - 3)^2 + [y - (-1)]^2 = 4^2$$

The center is  $(\underline{3}, \underline{-1})$  and the radius is  $\underline{4}$ . To graph the circle, place the point of a compass at  $(\underline{3}, \underline{-1})$ , set the radius at  $\underline{4}$  units, and swing the compass to draw a full circle.



✓ **Checkpoint** Complete the following exercises.

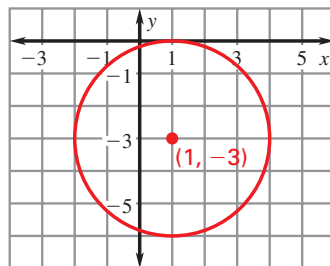
1. Write the standard equation of a circle with center  $(-5, -3)$  and radius 5.2.

$$(x + 5)^2 + (y + 3)^2 = 27.04$$

2. The point  $(4, -5)$  is on a circle whose center is  $(-2, 3)$ . Write the standard equation of the circle.

$$(x + 2)^2 + (y - 3)^2 = 100$$

3. Graph the equation  $(x - 1)^2 + (y + 3)^2 = 9$ .



**Example 4****Applying Graphs of Circles**

**Skydiving** A skydiving instruction school has a field with multiple landing targets. Each target is circle shaped. A coordinate plane is used to arrange the targets in the field, with the corner of the field as the origin. The equation  $(x - 8)^2 + (y - 4)^2 = 9$  represents one of the targets.

- Graph the landing target.
- The landing spots by the following skydivers are located as follows: Marika is at  $(7, 3)$ , Alex is at  $(3, 4)$ , Julia is at  $(10, 7)$ , and Caleb is at  $(9, 6)$ . Which skydivers landed on the target?

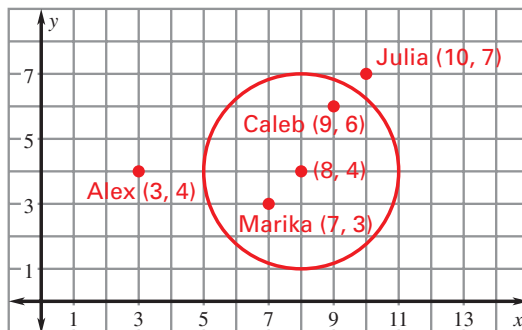
**Solution**

- Rewrite the equation to find the center and radius:

$$(x - 8)^2 + (y - 4)^2 = 9$$

$$(x - 8)^2 + (y - 4)^2 = \underline{3}^2$$

The center is  $(\underline{8}, \underline{4})$  and the radius is  $\underline{3}$ . Graph the circle below.



- Graph the landing spots of the skydivers. The graph shows that Marika and Caleb both landed on the target.