11.1 Angle Measures in Polygons

Goals
- Find the measures of interior and exterior angles of polygons.
- Use measures of angles of polygons to solve problems.

THEOREM 11.1: POLYGON INTERIOR ANGLES THEOREM

The sum of the measures of the interior angles of a convex n-gon is

$(n - \underline{2}) \cdot \underline{180}^\circ$.

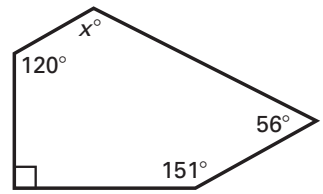
COROLLARY TO THEOREM 11.1

The measure of each interior angle of a regular n-gon is

$\frac{1}{\boxed{n}} \cdot (n - \underline{2}) \cdot \underline{180}^\circ$, or $\frac{\left(n - \boxed{2}\right) \cdot \boxed{180^\circ}}{\boxed{n}}$.

Example 1 *Finding Measures of Interior Angles of Polygons*

Find the value of x in the diagram.



Solution

The sum of the measures of the interior angles of any pentagon is $(\underline{5} - \underline{2}) \cdot \underline{180}^\circ = \underline{3} \cdot \underline{180}^\circ = \underline{540}^\circ$.

Add the measures of the interior angles of the pentagon.

$120^\circ + 90^\circ + 151^\circ + 56^\circ + x^\circ = \underline{540}^\circ$ **The sum is** $\underline{540}^\circ$.

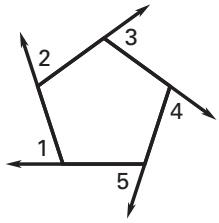
$\underline{417} + x = \underline{540}$ **Simplify.**

$x = \underline{123}$ **Subtract** $\underline{417}$ **from each side.**

Answer The measure of the fifth interior angle of the pentagon is $\underline{123}^\circ$.

THEOREM 11.2: POLYGON EXTERIOR ANGLES THEOREM

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.



COROLLARY TO THEOREM 11.2

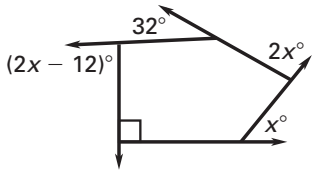
The measure of each exterior angle of a regular n-gon is

$\frac{1}{n} \cdot 360°$, or $\frac{360°}{n}$.

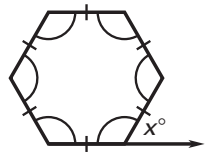
Example 2 *Finding the Measure of an Exterior Angle*

Find the value of x.

a.



b.



Solution

a. $90° + x° + 2x° + 32° + (2x - 12)° = 360°$ **Use Theorem 11.2.**

$5x + 110 = 360$ **Simplify.**

$5x = 250$ **Subtract.**

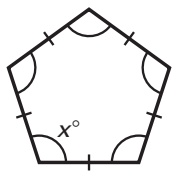
$x = 50$ **Divide.**

b. $x° = \frac{1}{6} \cdot 360°$ **Use $n = 6$ in the Corollary to Theorem 11.2.**

$= 60$ **Simplify.**

✔ ***Checkpoint*** **Find the value of x.**

1.



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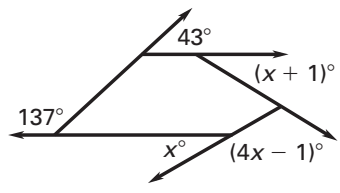
2.



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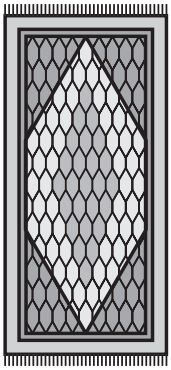
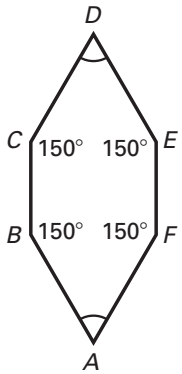
Example 3 *Finding Angle Measures of a Polygon*

Rugs The rug shown at the right contains a hexagonal design. Four of the angles in the hexagon have a measure of 150°. The other two angles are congruent. What is the measure of each angle?



Solution

Draw a sketch The diagram shows a sketch of the design. The design is a nonregular hexagon. $\angle B$, $\angle C$, $\angle E$, and $\angle F$ each measure 150°, and $\angle A$ is congruent to $\angle$ D. The sum of the measures of the interior angles of the hexagon is 720°.



Verbal Model

Sum of measures of interior angles	=	4	•	Measure of each obtuse angle	+
2	•	Measure of $\angle A$ and $\angle D$			

Labels

Sum of measures of interior angles = 720 **(degrees)**

Measure of each obtuse angle = 150 **(degrees)**

Measure of $\angle A$ and $\angle D = x$ **(degrees)**

Reasoning

$720 = 4 \cdot 150 + 2x$	**Write an equation.**
$720 = 600 + 2x$	**Simplify.**
$120 = 2x$	**Subtract 600 from each side.**
$60 = x$	**Divide each side by 2.**

Answer The measure of each of the two congruent angles is 60°.