11.2 Areas of Regular Polygons

Goals
- Find the area of an equilateral triangle.
- Find the area of a regular polygon.

VOCABULARY

Center of a polygon The center of a polygon is the center of its circumscribed circle.

Radius of a polygon The radius of a polygon is the radius of its circumscribed circle.

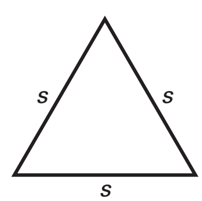
Apothem of a polygon The apothem of a polygon is the distance from the center of the polygon to any side of the polygon.

Central angle of a regular polygon A central angle of a regular polygon is an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon.

THEOREM 11.3: AREA OF AN EQUILATERAL TRIANGLE

The area of an equilateral triangle is one fourth the square of the length of the side times $\sqrt{3}$.

$A = \frac{1}{4}\sqrt{3}s^2$



Example 1 ***Finding the Area of an Equilateral Triangle***

Find the area of an equilateral triangle with 12 inch sides.

Solution

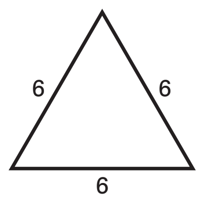
Use $s = 12$ in the formula from Theorem 11.3.

$$A = \frac{1}{4}\sqrt{3}s^2 = \frac{1}{4}\sqrt{3}(\underline{12}^2) = \frac{1}{4}(\underline{144})\sqrt{3} = \underline{36\sqrt{3}} \text{ in.}^2$$

Answer The area is $\underline{36\sqrt{3}}$ square inches, or about $\underline{62.4}$ square inches.

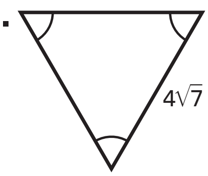
Checkpoint **Find the area of the triangle.**

1.



$9\sqrt{3} \approx 15.6$

2.



$28\sqrt{3} \approx 48.5$

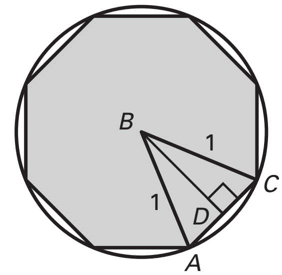
THEOREM 11.4: AREA OF A REGULAR POLYGON

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P.

$A = \frac{1}{2}\underline{aP}$ or $A = \frac{1}{2}\underline{a} \cdot \underline{ns}$

Example 2 *Finding the Area of a Regular Polygon*

A regular octagon is inscribed in a circle with radius 1 unit. Find the area of the octagon.



Solution

To apply the formula for the area of a regular octagon, find its apothem and perimeter.

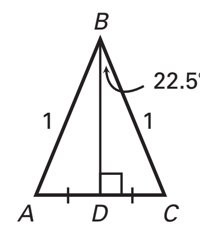
To find the measure of the central angle of a regular polygon, divide 360° by its number of sides.

The measure of central $\angle ABC$ is 45°.

In isosceles triangle $\triangle ABC$, the altitude BD bisects $\angle ABC$ and side $\overline{AC}$. So, $m\angle DBC =$ 22.5°. In right triangle $\triangle BDC$, you can use trigonometric ratios to find the lengths of the legs.

$$\cos 22.5^\circ = \frac{BD}{BC} = \frac{BD}{1} = BD$$

$$\sin 22.5^\circ = \frac{CD}{BC} = \frac{CD}{1} = CD$$



So, the apothem is $a = BD =$ cos 22.5°.

The perimeter is $P = 8(AC) = 8(2 \cdot CD) =$ 16 sin 22.5°.

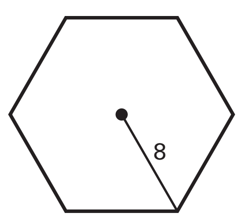
The area of the octagon is

$$A = \frac{1}{2}aP = \frac{1}{2}(\cos 22.5^\circ)(16 \sin 22.5^\circ) \approx 2.83.$$

Answer The area of the octagon is about 2.83 square units.

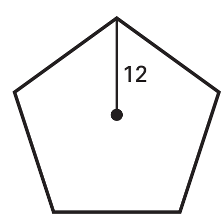
Checkpoint Find the area of the regular polygon.

3.



$96\sqrt{3} \approx 166.28$ sq. units

4.



$720 \cos 36^\circ \sin 36^\circ \approx$ 342.38 sq. units