11.3 Perimeters and Areas of Similar Figures

Goals • Compare perimeters and areas of similar figures.
• Use perimeters and areas of similar figures to solve real-life problems.

THEOREM 11.5: AREAS OF SIMILAR POLYGONS

If two polygons are similar with the lengths of corresponding sides in the ratio of $a : b$, then the ratio of their areas is $\underline{a^2} : \underline{b^2}$.

$$\frac{\text{Side length of Quad. I}}{\text{Side length of Quad. II}} = \frac{a}{b}$$

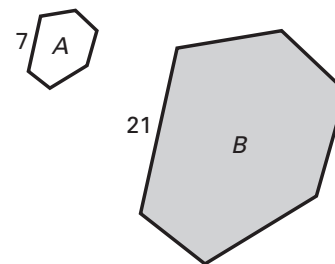$$\frac{\text{Area of Quad. I}}{\text{Area of Quad. II}} = \frac{a^2}{b^2}$$

Quad. I ~ Quad. II

Example 1 *Finding Ratios of Similar Polygons*

Hexagons *A* and *B* are similar.

a. Find the ratio (unshaded to shaded) of the perimeters of the hexagons.

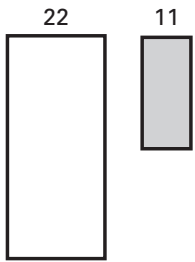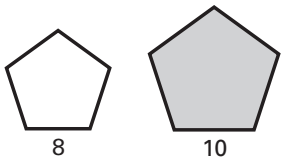**b.** Find the ratio (unshaded to shaded) of the areas of the hexagons.

Solution

The ratio of the lengths of corresponding sides in the hexagons is

$$\frac{7}{21} = \frac{1}{3}, \text{ or } \underline{1} : \underline{3}.$$

a. The ratio of the perimeters is also $\underline{1} : \underline{3}$. So, the perimeter of hexagon *A* is __one third__ the perimeter of hexagon *B*.

b. Using Theorem 11.5, the ratio of the area is $\underline{1^2} : \underline{3^2}$, or $\underline{1} : \underline{9}$. So, the area of hexagon *A* is __one ninth__ the area of hexagon *B*.

✔ *Checkpoint* **The polygons are similar. Find the ratio (unshaded to shaded) of their perimeters and of their areas.**

1.

22   11

Perimeter: 2 : 1

Area: 4 : 1

2.

8   10

Perimeter: 4 : 5

Area: 16 : 25

Example 2 *Using Areas of Similar Figures*

Comparing Costs You want to carpet a room that measures 40 feet by 48 feet. An advertisement states that the cost to carpet a room that measures 10 feet by 12 feet is $216. What is a reasonable cost for the larger room?

Solution

The ratio of the lengths of corresponding sides are equal. The ratio of the side lengths is $1 : \underline{4}$. So, the ratio of the areas is $1^2 : \underline{4^2}$, or $1 : \underline{16}$.

Because the cost of the carpet should be a function of its area, the carpet for the larger room should cost about $\underline{\text{sixteen}}$ times that of the smaller room.

$\underline{16} \times 216 = \underline{3456}$

Answer A reasonable cost for the larger room is $\underline{3456}$.

Example 3 *Finding Perimeters and Areas of Similar Polygons*

Gazebos A park has two gazebos. The floor of each gazebo has the shape of a regular hexagon. The floor of one gazebo has a perimeter of 72 feet and an area of about 374.12 square feet. The floor of the larger gazebo has a perimeter of 96 feet. Find its area.

Solution

All regular hexagons are similar because all corresponding angles are congruent and the corresponding side lengths are proportional.

Find the ratio of the perimeters of the two gazebos. This ratio is the same as the ratio of their side lengths.

$$\frac{\text{Perimeter of smaller gazebo}}{\text{Perimeter of larger gazebo}} = \frac{72}{\boxed{96}} = \frac{3}{4}$$

Calculate the area of the larger gazebo. Let *A* represent the area of the larger gazebo. The ratio of the area of the smaller gazebo to the area of the larger gazebo is $\underline{3}^2 : \underline{4}^2$, or $\underline{9} : \underline{16}$.

$$\frac{9}{16} = \frac{\boxed{374.12}}{A} \qquad \text{Write proportion.}$$

$$\underline{9} \cdot A = \underline{16} (\underline{374.12}) \qquad \text{Cross product property}$$

$$A = \frac{16(374.12)}{9} \qquad \text{Divide each side by } \underline{9}.$$

$$A \approx \underline{665.1} \qquad \text{Use a calculator.}$$

Answer The area of the larger gazebo is about $\underline{665.1}$ square feet.

✔ *Checkpoint* Complete the following exercise.

3. In Example 3, assume that the floor of the larger gazebo has a perimeter of 120 feet. Find its area.

about 1039.2 square feet