11.4 Circumference and Arc Length

Goals
- Find the circumference of a circle and the length of a circular arc.
- Use circumference and arc length to solve problems.

VOCABULARY

Circumference The circumference of a circle is the distance around the circle.

Arc length An arc length is a portion of the circumference of a circle.

THEOREM 11.6: CIRCUMFERENCE OF A CIRCLE

The circumference C of a circle is $C = \underline{\pi d}$ or $C = \underline{2\pi r}$, where d is the diameter of the circle and r is the radius of the circle.

Example 1 ***Using Circumference***

a. Find the circumference of a circle with radius 9 inches.

b. Find the diameter of a circle with a circumference of 58 inches.

Solution

a. $C = 2\pi r$

$= 2 \cdot \pi \cdot \underline{9}$

$= \underline{18}\,\pi$

$\approx \underline{56.55}$

So, the circumference is about $\underline{56.55}$ inches.

b. $C = \pi d$

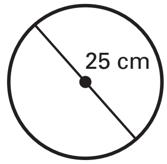
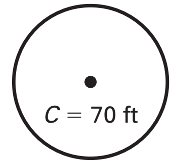
$\underline{58} = \pi d$

$\dfrac{\boxed{58}}{\pi} = d$

$\underline{18.46} \approx d$

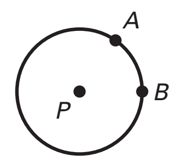
So, the diameter is about $\underline{18.46}$ inches.

Checkpoint Find the indicated measure.

1. Circumference	2. Radius
 about 78.54 cm	 about 11.14 ft

ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



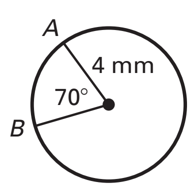
$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or}$$

$$\text{Arc length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$$

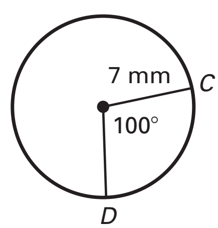
Example 2 *Finding Arc Lengths*

Find the length of each arc.

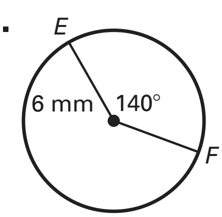
a.



b.



c.



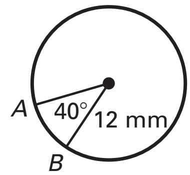
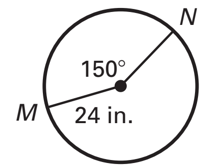
Solution

a. Arc length of $\overset{\frown}{AB} = \frac{70°}{360°} \cdot 2\pi(4) \approx 4.89$ millimeters

b. Arc length of $\overset{\frown}{CD} = \frac{100°}{360°} \cdot 2\pi(7) \approx 12.22$ millimeters

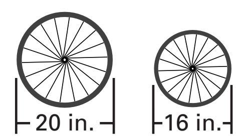
c. Arc length of $\overset{\frown}{EF} = \frac{140°}{360°} \cdot 2\pi(6) \approx 14.66$ millimeters

✔ *Checkpoint* **Find the indicated measure.**

3. Length of $\overset{\frown}{AB}$	**4.** Length of $\overset{\frown}{MN}$
	
about 8.38 mm	about 62.83 in.

Example 3 ***Using Circumference***

Tricycles The diagram at the right shows two tires from a tricycle. How many revolutions does each tire make while traveling 250 feet? Round answers to one decimal place.



Solution

The larger tire has a diameter of _20_ inches. Its circumference is $\pi \cdot$ _20_, or about _62.83_ inches.

The smaller tire has a diameter of _16_ inches. Its circumference is $\pi \cdot$ _16_, or about _50.27_ inches.

To find the number of revolutions made, divide the distance the tricycle travels by the tire circumference.

Larger tire: $\dfrac{250 \text{ ft}}{\boxed{62.83} \text{ in.}} = \dfrac{\boxed{3000} \text{ in.}}{\boxed{62.83} \text{ in.}}$ **Convert feet to inches.**

$\approx$ _47.7_ **Simplify.**

Smaller tire: $\dfrac{250 \text{ ft}}{\boxed{50.27} \text{ in.}} = \dfrac{\boxed{3000} \text{ in.}}{\boxed{50.27} \text{ in.}}$ **Convert feet to inches.**

$\approx$ _59.7_ **Simplify.**

Answer The larger tire makes about _47.7_ revolutions in 250 feet and the smaller tire makes about _59.7_ revolutions in 250 feet.