11.5 Areas of Circles and Sectors

Goals
- Find the area of a circle and a sector of a circle.
- Use areas of circles and sectors to solve problems.

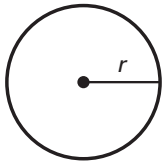
VOCABULARY

Sector of a circle A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.

THEOREM 11.7: AREA OF A CIRCLE

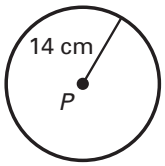
The area of a circle is π times the square of the radius.

$A = \pi r^2$

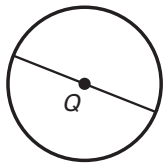


Example 1 *Using the Area of a Circle*

a. Find the area of $\odot P$.



b. Find the diameter of $\odot Q$.



Area = 125 in.^{2}

Solution

a. Use $r =$ 14 in the area formula.

$A = \pi r^2$

$= \pi \cdot$ 14 ^{2}

$=$ 196 π

$\approx$ 615.75

The area is 196 π, or about 615.75, square centimeters.

b. The diameter is twice the radius.

$A = \pi r^2$

125 $= \pi r^2$

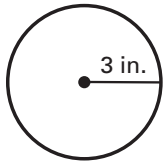
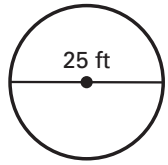
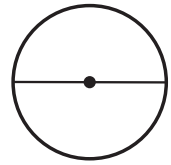
$\frac{125}{\pi} = r^2$

39.79 $\approx r^2$

6.31 $\approx r$

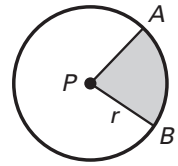
The diameter is about 2 (6.31), or 12.62, inches.

Checkpoint Find the indicated measure.

1. Area	2. Area	3. Diameter
		 Area = 248 mm^2
about 28.27 in.^{2}	about 490.87 ft^2	about 17.77 mm

THEOREM 11.8: AREA OF A SECTOR

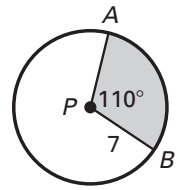
The ratio of the area A of a sector of a circle to the area of the circle is equal to the ratio of the measure of the intercepted arc to 360°.



$$\frac{A}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360^\circ}, \text{ or } A = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$$

Example 2 *Finding the Area of a Sector*

Find the area of the sector.



Solution

Sector APB intercepts an arc whose measure is 110°.

The radius is 7 units.

$$A = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$$ **Formula for area of a sector**

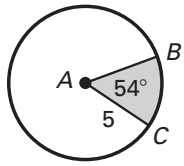
$$= \frac{110^\circ}{360^\circ} \cdot \pi(7^2)$$ **Substitute known values.**

$$\approx 47.04$$ **Use a calculator.**

Answer The area of the sector is about 47.04 square units.

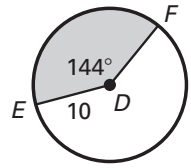
Checkpoint Find the area of the shaded region.

4.



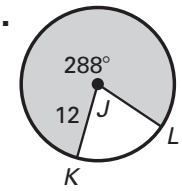
about 11.78 sq. units

5.



about 125.66 sq. units

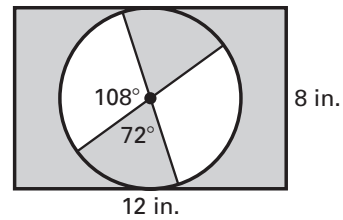
6.



about 361.91 sq. units

Example 3 *Finding the Area of a Region*

Find the area of the shaded region.



Solution

The shaded region consists of a rectangle and two sectors of a circle.

The rectangle has a length of 12 inches and a width of 8 inches. The radius of the circle is 4 inches.

$$\boxed{\text{Area}} = \boxed{\text{Area of rectangle}} - 2 \cdot \boxed{\text{Area of one unshaded sector}}$$

$$= 12 \cdot 8 - 2 \cdot \frac{\boxed{108^\circ}}{360^\circ} \cdot \pi \cdot 4^2$$

$$= 96 - 2\left(\frac{\boxed{3}}{10} \cdot \pi \cdot 16\right)$$

$$= 96 - \frac{\boxed{48}}{5}\pi$$

$$\approx 65.84$$

Answer The area of the shaded region is about 65.84 square inches.