12.1 Exploring Solids

Goals
- Use properties of polyhedra.
- Use Euler's Theorem.

VOCABULARY

Polyhedron A polyhedron is a solid that is bounded by polygons that enclose a single region of space.

Face The faces of a polyhedron are polygons.

Edge An edge of a polyhedron is a line segment formed by the intersection of two faces of the polyhedron.

Vertex A vertex of a polyhedron is a point where three or more edges of the polyhedron meet.

Regular polyhedron A regular polyhedron is a polyhedron whose faces are all congruent regular polygons.

Convex polyhedron A convex polyhedron is a polyhedron such that any two points on its surface can be connected by a line segment that lies entirely inside or on the polyhedron.

Cross section A cross section is the intersection of a plane and a solid.

Platonic solids A Platonic solid is one of five regular polyhedra: a regular tetrahedron, a cube, a regular octahedron, a regular dodecahedron, and a regular icosahedron. These solids are named after Plato, a Greek mathematician and philosopher.

Tetrahedron A tetrahedron is a polyhedron with four faces.

Octahedron An octahedron is a polyhedron with eight faces.

Dodecahedron A dodecahedron is a polyhedron with twelve faces.

Icosahedron An icosahedron is a polyhedron with twenty faces.

TYPES OF SOLIDS

Of the five solids below, the prism and pyramid are polyhedra. The cylinder, sphere, and cone are not polyhedra.


Prism


Cylinder

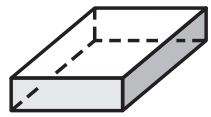

Pyramid

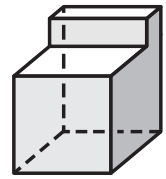

Sphere

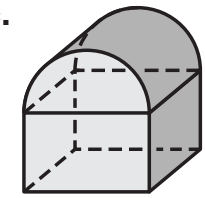

Cone

Example 1 *Identifying Polyhedra*

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.

a. 

b. 

c. 

Solution

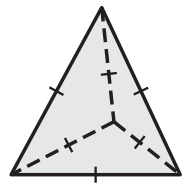
a. This is a polyhedron. It has 6 faces, 8 vertices, and 12 edges.

b. This is a polyhedron. It has 8 faces, 12 vertices, and 18 edges.

c. This is not a polyhedron. Some of the faces are not polygons.

Example 2 *Classifying Polyhedra*

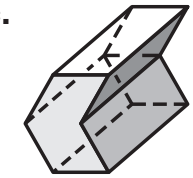
Is the polyhedron convex? Is it regular?

a. 

convex, regular

b. 

convex, nonregular

c. 

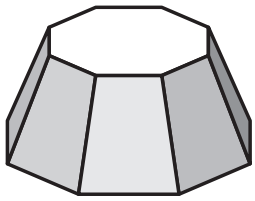
nonconvex, nonregular

THEOREM 12.1: EULER'S THEOREM

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E +$ 2 .

Example 3 *Using Euler's Theorem*

The solid has 10 faces: 8 trapezoids and 2 octagons. How many vertices does the solid have?



On their own, 8 trapezoids and 2 octagons have 8(4) + 2(8) = 48 sides. In the solid, each side is shared by exactly two polygons. So the number of edges is 24 . Use Euler's Theorem to find the number of vertices.

$F + V = E + 2$ **Write Euler's Theorem.**

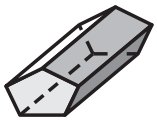
10 $+ V =$ 24 $+ 2$ **Substitute.**

$V =$ 16 **Solve for V.**

Answer The solid has 16 vertices.

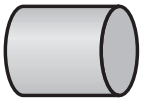
✔ ***Checkpoint*** **Is the solid a polyhedron? If so, is it convex? Is it regular?**

1.



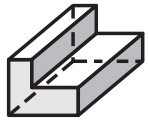
Yes, convex; nonregular

2.



No

3.



Yes, nonconvex; nonregular

4. Critical Thinking Is it possible for a polyhedron to have 16 faces, 34 vertices, and 50 edges? Explain.

No; From Euler's Theorem, the number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.