12.3 Surface Area of Pyramids and Cones

Goal • Find the surface area of a pyramid and of a cone.

VOCABULARY

Pyramid A pyramid is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex. The intersection of two lateral faces is a lateral edge. The intersection of the base and a lateral face is a base edge. The altitude, or height, of the pyramid is the perpendicular distance between the base and the vertex.

Regular pyramid A regular pyramid has a regular polygon for a base and its height meets the base at its center. The slant height of a regular pyramid is the altitude of any lateral face. A nonregular pyramid does not have a slant height.

Circular cone or cone A circular cone, or cone, has a circular base and a vertex that is not in the same plane as the base. The altitude, or height, is the perpendicular distance between the vertex and the base.

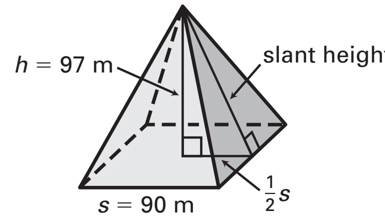
Right cone In a right cone, the height meets the base at its center and the slant height is the distance between the vertex and a point on the base edge.

Lateral surface of a cone The lateral surface of a cone consists of all segments that connect the vertex with points on the base edge.

A *regular pyramid* is considered a regular polyhedron only if *all* its faces, including the base, are congruent. So, the only pyramid that is a regular polyhedron is the regular triangular pyramid, or *tetrahedron*.

Example 1 ***Finding the Area of a Lateral Face***

Find the area of each lateral face of the regular pyramid shown at the right.



Solution

To find the slant height of the pyramid, use the Pythagorean Theorem.

$$(\text{Slant height})^2 = h^2 + \left(\frac{1}{2}s\right)^2 \qquad \textbf{Write formula.}$$

$$(\text{Slant height})^2 = \underline{\ 97\ }^2 + \underline{\ 45\ }^2 \qquad \textbf{Substitute.}$$

$$(\text{Slant height})^2 = \underline{\ 11{,}434\ } \qquad \textbf{Simplify.}$$

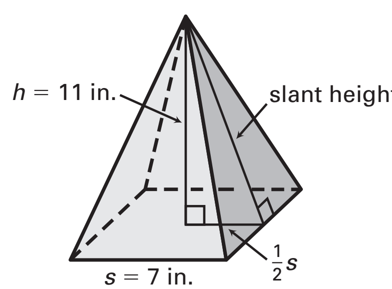
$$\text{Slant height} = \underline{\ \sqrt{11{,}434}\ } \qquad \textbf{Take the positive square root.}$$

$$\text{Slant height} \approx \underline{\ 106.93\ } \qquad \textbf{Use a calculator.}$$

Answer So, the area of each lateral face is $\frac{1}{2}$(base of lateral face)(slant height), or about $\frac{1}{2}(\underline{\ 90\ })(\underline{\ 106.93\ })$, which is about $\underline{\ 4812\ }$ square meters.

✔ ***Checkpoint*** **Complete the following exercise.**

1. Find the area of a lateral face of the regular pyramid. Round the result to one decimal place.

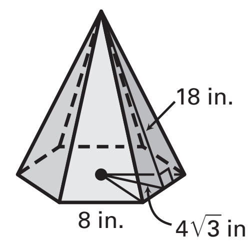


40.4 in.^{2}

THEOREM 12.4: SURFACE AREA OF A REGULAR PYRAMID

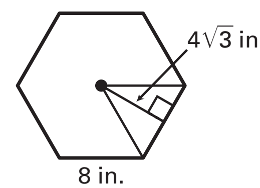
The surface area S of a regular pyramid is $S = B + \underline{\frac{1}{2}Pl}$, where B is the area of the base, P is the perimeter of the base, and l is the slant height.

Example 2 ***Finding the Surface Area of a Pyramid***



To find the surface area of the regular pyramid shown, start by finding the area of the base.

Use the formula for the area of a regular polygon, $\frac{1}{2}$(apothem)(perimeter).



A diagram of the base is shown at the right. After substituting, the area of the base is $\frac{1}{2}(4\sqrt{3})(6 \cdot \underline{8})$, or $\underline{96}\sqrt{3}$ square inches.

Now you can find the surface area, using $\underline{96}\sqrt{3}$ for the area of the base, B.

$S = B + \frac{1}{2}Pl$	**Write formula.**
$= \underline{96}\sqrt{3} + \frac{1}{2}(\underline{48})(\underline{18})$	**Substitute.**
$= \underline{96}\sqrt{3} + \underline{432}$	**Simplify.**
$\approx \underline{598.3}$	**Use a calculator.**

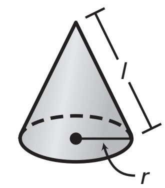
Answer So, the surface area is about $\underline{598.3}$ square inches.

THEOREM 12.5: SURFACE AREA OF A RIGHT CONE

The surface area S of a right cone is

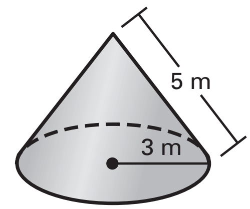
$S = \pi r^2 + \underline{\pi r l}$,

where r is the radius of the base and l is the slant height.



Example 3 ***Finding the Surface Area of a Right Cone***

To find the surface area of the right cone shown, use the formula for the surface area.

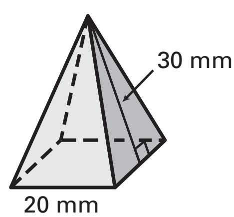


$S = \pi r^2 + \pi r l$	**Write formula.**
$= \pi(\underline{3})^2 + \pi(\underline{3})(\underline{5})$	**Substitute.**
$= \underline{9}\,\pi + \underline{15}\,\pi$	**Simplify.**
$= \underline{24}\,\pi$	**Add.**

Answer The surface area is $\underline{24}\,\pi$ square meters, or about $\underline{75.4}$ square meters.

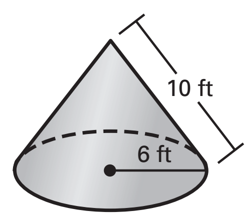
Checkpoint **Find the surface area of the solid. Round your result to two decimal places.**

2. Regular pyramid



1600 mm^2

3. Right cone



301.60 ft^2