12.4 Volume of Prisms and Cylinders

Goals
- Use volume postulates.
- Find the volumes of prisms and cylinders.

VOCABULARY

Volume of a solid The volume of a solid is the number of cubic units contained in the solid's interior.

POSTULATE 27: VOLUME OF A CUBE

The volume of a cube is the cube of the length of its side, or $V =$ s^3.

POSTULATE 28: VOLUME CONGRUENCE POSTULATE

If two polyhedra are congruent, then they have the same volume.

POSTULATE 29: VOLUME ADDITION POSTULATE

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

THEOREM 12.6: CAVALIERI'S PRINCIPLE

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

THEOREM 12.7: VOLUME OF A PRISM

The volume V of a prism is $V =$ Bh, where B is the area of a base and h is the height.

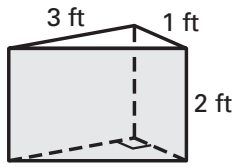
THEOREM 12.8: VOLUME OF A CYLINDER

The volume V of a cylinder is $V = Bh =$ $\pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

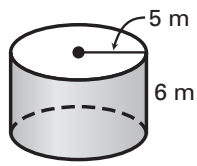
Example 1 ***Finding Volumes***

Find the volume of the right prism and the right cylinder.

a.



b.



Solution

a. The area B of the base is $\frac{1}{2}$(1)(3), or $\frac{3}{2}$ ft^2. Use $h = 2$ to find the volume.

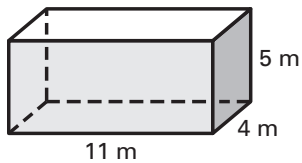
$V = Bh = \frac{3}{2}$ (2) = 3 ft^3

b. The area B of the base is $\pi \cdot$ 5 ^{2}, or 25 π m^2. Use $h = 6$ to find the volume.

$V = Bh =$ 25 π(6) = 150 $\pi \approx$ 471.24 m^3

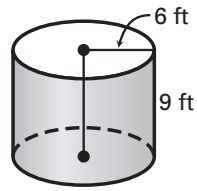
✔ ***Checkpoint*** **Find the volume of the solid. Round your result to two decimal places.**

1. Right prism



220 m^3

2. Right cylinder

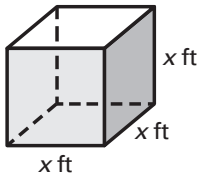


1017.88 ft^3

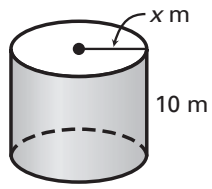
Example 2 *Using Volumes*

Use the measurements given to solve for x.

a. Cube, $V = 90$ ft^3



b. Right cylinder, $V = 1253$ m^3



Solution

a. A side length of the cube is x feet.

$V = s^3$		**Formula for volume of cube**
90 $= x^3$		**Substitute.**
4.48 $\approx x$		**Take the cube root.**

Answer So, the height, width, and length of the cube are about 4.48 feet.

b. The area of the base is πx^2 square meters.

$V = Bh$	**Formula for volume of cylinder**
1253 $= \pi x^2($ 10 $)$	**Substitute.**
1253 $=$ 10 πx^2	**Rewrite.**
$\frac{1253}{10\ \pi} = x^2$	**Divide each side by** 10 π.
39.88 $\approx x^2$	**Simplify.**
6.32 $\approx x$	**Find the positive square root.**

Answer So, the radius of the cylinder is about 6.32 meters.