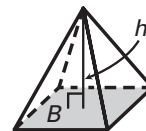# 12.5 Volume of Pyramids and Cones

Goal • Find the volume of pyramids and cones.

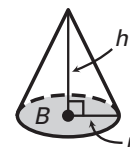---

THEOREM 12.9: VOLUME OF A PYRAMID

The volume V of a pyramid is $V = \dfrac{1}{3}Bh$, where

B is the area of the base and h is the height.

THEOREM 12.10: VOLUME OF A CONE

The volume V of a cone is $V = \dfrac{1}{3}Bh = \dfrac{1}{3}\pi r^2 h$,

where B is the area of the base, h is the height, and r is the radius of the base.

Example 1   *Finding the Volume of a Pyramid*

Find the volume of the pyramid with the regular base.
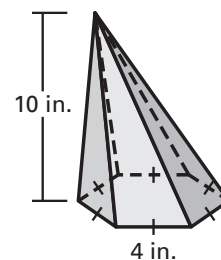
Solution

The base can be divided into six equilateral triangles. Using the formula for the area of an equilateral triangle, $\dfrac{1}{4}\sqrt{3} \cdot s^2$, the area of the base B can be found as follows:

$$\underline{6} \cdot \dfrac{1}{4}\sqrt{3} \cdot s^2 = \underline{6} \cdot \dfrac{1}{4}\sqrt{3} \cdot \underline{4}^2 = \underline{24}\sqrt{3} \text{ in.}^2$$

Use Theorem 12.9 to find the volume of the pyramid.

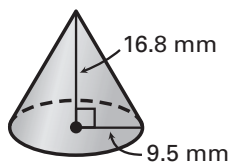$$V = \dfrac{1}{3}Bh = \dfrac{1}{3}(\underline{24\sqrt{3}})(\underline{10}) = \underline{80}\sqrt{3}$$

Answer The volume of the pyramid is $\underline{80}\sqrt{3}$, or about $\underline{138.6}$ cubic inches.
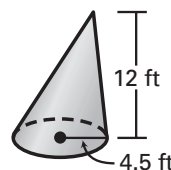
10 in.

4 in.

4 in.

Example 2 *Finding the Volume of a Cone*

Find the volume of each cone.

a. Right circular cone

b. Oblique circular cone

16.8 mm

9.5 mm

12 ft

4.5 ft

Solution

a. Use the formula for the volume of a cone.

$V = \dfrac{1}{3}Bh$ **Formula for volume of cone**

$= \dfrac{1}{3}(\pi r^2)h$ **Base area equals πr^2.**

$= \dfrac{1}{3}(\pi \underline{\,9.5\,}{}^2)\,\underline{16.8}$ **Substitute.**

$= \underline{\,505.4\,}\ \pi$ **Simplify.**

Answer The volume of the cone is $\underline{\,505.4\,}\ \pi$, or about $\underline{\,1588\,}$ cubic millimeters.

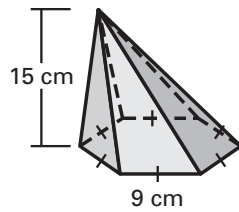**b.** Use the formula for the volume of a cone.

$V = \dfrac{1}{3}Bh$ **Formula for volume of cone**

$= \dfrac{1}{3}(\pi r^2)h$ **Base area equals πr^2.**

$= \dfrac{1}{3}(\pi \underline{\,4.5\,}{}^2)\,\underline{12}$ **Substitute.**

$= \underline{\,81\,}\ \pi$ **Simplify.**

Answer The volume of the cone is $\underline{\,81\,}\ \pi$, or about $\underline{\,254\,}$ cubic feet.
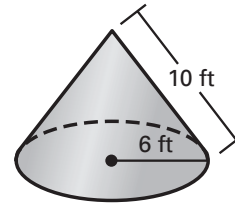
✔ *Checkpoint* **Find the volume of the solid. Round your result to two decimal places.**

1. Pyramid with regular base

15 cm

9 cm

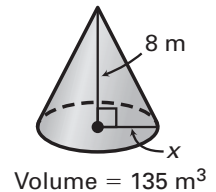1052.22 cm³

2. Right circular cone

10 ft

6 ft

301.60 ft³

Example 3   *Using the Volume of a Cone*

Use the given measurements to solve for x.

8 m

x

Volume = 135 m³

Solution

$$V = \frac{1}{3}(\pi r^2)h \qquad \text{Formula for volume}$$

$$\underline{135} = \frac{1}{3}(\pi x^2)(\underline{8}) \qquad \text{Substitute.}$$

$$\underline{405} = \underline{8}\,\pi x^2 \qquad \text{Multiply each side by } \underline{3}.$$

$$\underline{16.11} \approx x^2 \qquad \text{Divide each side by } \underline{8\pi}.$$

$$\underline{4.01} \approx x \qquad \text{Find positive square root.}$$

Answer The radius of the cone is about $\underline{4.01}$ meters.