12.6 Surface Area and Volume of Spheres

Goals
- Find the surface area of a sphere.
- Find the volume of a sphere.

VOCABULARY

Sphere A sphere is the locus of points in space that are a given distance from a point. The point is called the center of the sphere.

Radius of a sphere A radius of a sphere is a segment from the center to a point on the sphere.

Chord of a sphere A chord of a sphere is a segment whose endpoints are on the sphere.

Diameter of a sphere A diameter is a chord that contains the center of the sphere.

Great circle A great circle is the intersection of a sphere and a plane that contains the center of the sphere.

Hemisphere Half of a sphere, formed when a great circle separates a sphere into two congruent halves

THEOREM 12.11: SURFACE AREA OF A SPHERE

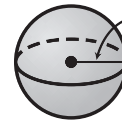
The surface area S of a sphere with radius r is $S = \underline{4\pi r^2}$.

Example 1 *Finding the Surface Area of a Sphere*

Find the surface area. When the radius doubles, does the surface area double?

a.

b.



Solution

a. $S = 4\pi r^2 = 4\pi(\underline{\ 3\ })^2 = \underline{\ 36\ }\pi \text{ cm}^2$

b. $S = 4\pi r^2 = 4\pi(\underline{\ 6\ })^2 = \underline{\ 144\ }\pi \text{ cm}^2$

The surface area of the sphere in part (b) is ___four___ times greater than the surface area of the sphere in part (a) because $\underline{\ 36\ }\pi \cdot \underline{\ 4\ } = \underline{\ 144\ }\pi$.

Answer When the radius of a sphere doubles, the surface area ___does not double___.

Example 2 *Using a Great Circle*

The circumference of a great circle of a sphere is 7.4π feet. What is the surface area of the sphere?

Solution

Begin by finding the radius of the sphere.

$C = 2\pi r$	**Formula for circumference of circle**
$\underline{\ 7.4\pi\ } = 2\pi r$	**Substitute for C.**
$\underline{\ 3.7\ } = r$	**Divide each side by 2π.**

Using a radius of ___3.7___ feet, the surface area is

$S = 4\pi r^2 = 4\pi(\underline{\ 3.7\ })^2 = \underline{\ 54.76\ }\pi \text{ ft}^2$.

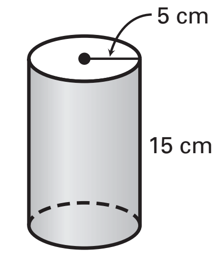
Answer The surface area of the sphere is $\underline{\ 54.76\ }\pi \text{ ft}^2$, or about ___172___ ft^2.

THEOREM 12.12: VOLUME OF A SPHERE

The volume V of a sphere with radius r is $V = \underline{\frac{4}{3}\pi r^3}$.

Example 3 *Finding the Volume of a Sphere*

What is the radius of a sphere made from the cylinder of modeling clay shown? Assume the sphere has the same volume as the cylinder.

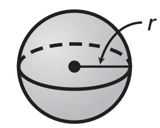


Cylinder of modeling clay

Solution

To find the volume of the cylinder of modeling clay, use the formula for the volume of a cylinder.

$V = \pi r^2 h = \pi(\underline{\ 5\ })^2(\underline{\ 15\ }) = \underline{\ 375\ }\pi \text{ cm}^3$



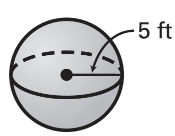
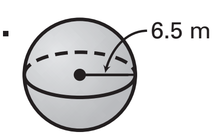
Sphere made from cylinder of modeling clay

To find the radius of the sphere, use the formula for the volume of a sphere and solve for *r*.

$V = \frac{4}{3}\pi r^3$		**Formula for volume of sphere**
$\underline{\ 375\ }\pi = \frac{4}{3}\pi r^3$		**Substitute for *V*.**
$\underline{\ 1125\ }\pi = 4\pi r^3$		**Multiply each side by $\underline{\ 3\ }$.**
$\underline{\ 281.25\ } = r^3$		**Divide each side by $\underline{\ 4\pi\ }$.**
$\underline{\ 6.55\ } \approx r$		**Use a calculator to take the cube root.**

Answer The radius of the sphere is about $\underline{\ 6.55\ }$ centimeters.

✔ ***Checkpoint*** **Find the surface area and volume of the sphere. Round your results to two decimal places.**

1.  314.16 ft^2; 523.60 ft^3	**2.**  530.93 m^2; 1150.35 m^3