12.7 Similar Solids

Goals
- Find and use the scale factor of similar solids.
- Use similar solids to solve real-life problems.

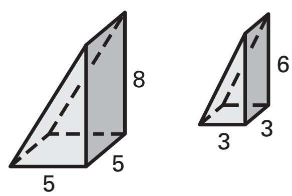
VOCABULARY

Similar solids Two solids with equal ratios of corresponding linear measures, such as heights or radii, are called similar solids.

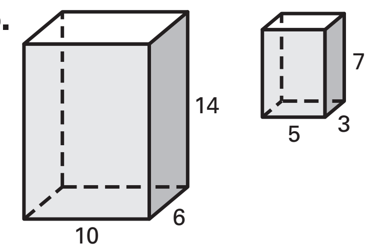
Example 1 *Identifying Similar Solids*

Decide whether the two solids are similar. If so, compare the volumes of the solids.

a.



b.



Solution

a. The solids are not similar because the ratios of corresponding linear measures are not equal, as shown.

lengths: $\frac{5}{3}$ widths: $\frac{5}{3}$ heights: $\frac{8}{6} = \frac{4}{3}$

b. The solids are similar because the ratios of corresponding linear measures are equal, as shown. The solids have a scale factor of 2 : 1.

lengths: $\frac{10}{5} = \frac{2}{1}$ widths: $\frac{6}{3} = \frac{2}{1}$ heights: $\frac{14}{7} = \frac{2}{1}$

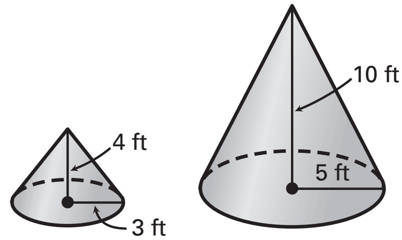
The volume of the larger prism is $V = Bh =$ 60 (14) = 840.

The volume of the smaller prism is $V = Bh =$ 15 (7) = 105.

The ratio of side lengths is 2 : 1 and the ratio of volumes is 840 : 105, or 8 : 1.

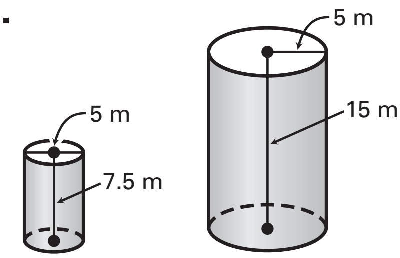
✔ *Checkpoint* **Decide whether the two solids are similar.**

1.



not similar

2.



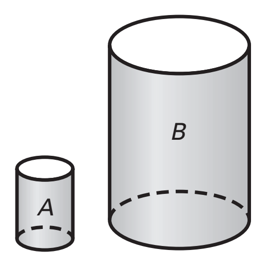
similar

THEOREM 12.13: SIMILAR SOLIDS THEOREM

If two similar solids have a scale factor of $a : b$, then corresponding areas have a ratio of $a^2 : b^2$, and corresponding volumes have a ratio of $a^3 : b^3$.

Example 2 ***Using the Scale Factor of Similar Solids***

Cylinders *A* and *B* are similar with a scale factor of 2 : 5. Find the surface area and volume of cylinder *B* given that the surface area of cylinder *A* is 96π square feet and the volume of cylinder *A* is 128π cubic feet.



Solution

Begin by using Theorem 12.13 to set up two proportions.

$$\frac{\text{Surface area of } A}{\text{Surface area of } B} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of } A}{\text{Volume of } B} = \frac{a^3}{b^3}$$

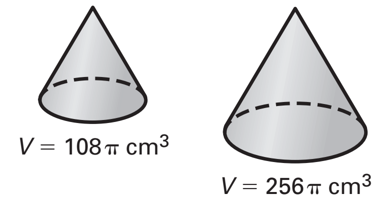
$$\frac{96\pi}{\text{Surface area of } B} = \frac{4}{25} \qquad \frac{128\pi}{\text{Volume of } B} = \frac{8}{125}$$

$$\text{Surface area of } B = 600\pi \qquad \text{Volume of } B = 2000\pi$$

Answer The surface area of cylinder *B* is 600π square feet and the volume of cylinder *B* is 2000π cubic feet.

Example 3 *Finding the Scale Factor of Similar Solids*

The two cones are similar. Find the scale factor.



Solution

Find the ratio of the two volumes.

$$\frac{a^3}{b^3} = \frac{108\pi}{256\pi}$$ **Write ratio of volumes.**

$$\frac{a^3}{b^3} = \frac{27}{64}$$ **Simplify.**

$$\frac{a}{b} = \frac{3}{4}$$ **Find the cube root.**

Answer The two cones have a scale factor of 3 : 4.

Example 4 *Comparing Similar Solids*

Two punch bowls are similar with a scale factor of 2 : 3. The amount of concentrate to be added is proportional to the volume. How much concentrate does the smaller bowl require if the larger bowl requires 48 ounces?

Solution

Using the scale factor, the ratio of the volume of the smaller punch bowl to the larger punch bowl is

$$\frac{a^3}{b^3} = \frac{\boxed{2}^3}{\boxed{3}^3} = \frac{8}{27} \approx \frac{1}{\boxed{3.4}}.$$

The ratio of the volumes of the concentrates is about 1 : 3.4. The amount of concentrate for the smaller punch bowl can be found by multiplying the amount of concentrate for the larger punch bowl by $\frac{1}{\boxed{3.4}}$ as follows: $48\left(\frac{1}{\boxed{3.4}}\right) \approx 14.1$ ounces.

Answer The smaller bowl requires about 14.1 ounces of concentrate.