

2.3

Deductive Reasoning

- Goals**
- Use symbolic notation to represent logical statements.
 - Form conclusions by applying the laws of logic to true statements.

VOCABULARY

Logical argument A logical argument is an argument based on deductive reasoning, which uses facts, definitions, and accepted properties in a logical order.

Example 1 Using Symbolic Notation

Let p be “the value of x is 7” and let q be “ x is less than 10.”

- Write $p \rightarrow q$ in words.
- Write $q \rightarrow p$ in words.
- Decide whether the biconditional statement $p \leftrightarrow q$ is true.

Solution

- If the value of x is 7, then x is less than 10.
- If x is less than 10, then the value of x is 7.
- The conditional statement in part (a) is true. The converse in part (b) is false. So, the biconditional statement $p \leftrightarrow q$ is false.

Example 2 Writing an Inverse and a Contrapositive

Let p be “my favorite TV show is on” and let q be “it is 8:00 P.M.”

- Write the contrapositive of $p \rightarrow q$.
- Write the inverse of $p \rightarrow q$.

Solution

- Contrapositive: $\sim q \rightarrow \sim p$
If it is not 8:00 P.M., then my favorite TV show is not on.
- Inverse: $\sim p \rightarrow \sim q$
If my favorite TV show is not on, then it is not 8:00 P.M.

LAW OF DETACHMENT

If $p \rightarrow q$ is a true conditional statement and p is true, then q is true.

Example 3 Using the Law of Detachment

State whether the argument is valid.

- If Roger gets a part-time job, then he will buy a new bicycle. Roger buys a new bicycle. So, Roger got a part-time job.
- If two angles are vertical angles, then they are congruent. $\angle 1$ and $\angle 2$ are vertical angles. So, $\angle 1$ and $\angle 2$ are congruent.

Solution

- This logical argument implies that because Roger bought a new bicycle, he must have got a part-time job.

The argument is not a valid use of the Law of Detachment.

- The statement $p \rightarrow q$ is true and the hypothesis p is true. So, you can conclude that the conclusion q is true.

The argument is a valid use of the Law of Detachment.

- ✓ **Checkpoint** In Checkpoint Exercises 1–4, let p be “the stereo is playing” and let q be “I am wearing headphones.”

1. Write $p \rightarrow q$. If the stereo is playing, then I am wearing headphones.	2. Write $q \rightarrow p$. If I am wearing headphones, then the stereo is playing.
3. Write $\sim q \rightarrow \sim p$. If I am not wearing headphones, then the stereo is not playing.	4. Write $\sim p \rightarrow \sim q$. If the stereo is not playing, then I am not wearing headphones.
5. State whether the following argument is valid. If two adjacent angles form a straight angle, then the angles are supplementary. $\angle 1$ and $\angle 2$ are supplementary. So, you can conclude that $\angle 1$ and $\angle 2$ are adjacent. <u>not valid</u>	

LAW OF SYLLOGISM

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then $p \rightarrow r$ is true.

Example 4 Using the Law of Syllogism

Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a cat is the largest of all cats, then it can weigh 650 pounds.
2. If a cat lives in a pride, then it is a lion.
3. If a cat weighs 650 pounds, then it is a tiger.
4. If a cat is a tiger, then it hunts alone.
5. If a cat is a lion, then it can weigh 400 pounds.

Solution

Here are three conditional statements that use the Law of Syllogism.

- a. If a cat lives in a pride, then it can weigh 400 pounds.
- b. If a cat is the largest of all cats, then it is a tiger.
- c. If a cat is the largest of all cats, then it hunts alone.

✔ **Checkpoint** Complete the following exercise.

6. Write two conditional statements that can be made from the following true statements using the Law of Syllogism.
 - a. If an elephant lives in India or Southeast Asia, then the elephant is an Indian elephant.
 - b. If an elephant is an African elephant, then it can weigh up to 8 tons.
 - c. If an elephant is an Indian elephant, then it can weigh up to 3.5 tons.
 - d. If an elephant lives in Africa, then it is an African elephant.

If an elephant lives in India or Southeast Asia, then it can weigh up to 3.5 tons.

If an elephant lives in Africa, then it can weigh up to 8 tons.