2.5 Proving Statements about Segments

Goals • Justify statements about congruent segments.
• Write reasons for steps in a proof.

VOCABULARY

Theorem A theorem is a true statement that follows as a result of other true statements.

Two-column proof A two-column proof is a type of proof written as numbered statements and reasons that show the logical order of an argument.

Paragraph proof A paragraph proof is a type of proof written in paragraph form.

THEOREM 2.1 PROPERTIES OF SEGMENT CONGRUENCE

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 1 ***Transitive Property of Segment Congruence***

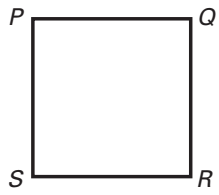
You can prove the Transitive Property of Segment Congruence as follows.

Given: $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$
Prove: $\overline{JK} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$	1. Given
2. $JK = MN$, $MN = PQ$	2. Definition of congruent segments
3. $JK = PQ$	3. Transitive property of equality
4. $\overline{JK} \cong \overline{PQ}$	4. Definition of congruent segments

Example 2 *Using Congruence*

Use the diagram and the given information to complete the proof.



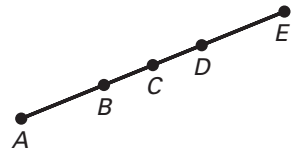
Given: $\overline{PQ} \cong \overline{RS}$, $\overline{PQ} \cong \overline{QR}$, $\overline{PS} \cong \overline{RS}$

Prove: $\overline{PS} \cong \overline{QR}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. Given
3. $\overline{RS} \cong \overline{QR}$	3. Transitive Property of Congruence
4. $\overline{PS} \cong \overline{RS}$	4. Given
5. $\overline{PS} \cong \overline{QR}$	5. Transitive Property of Congruence

Example 3 *Using Segment Relationships*

In the diagram, $AC = CE$ and $AB = DE$. Show that C is the midpoint of $\overline{BD}$.



Solution

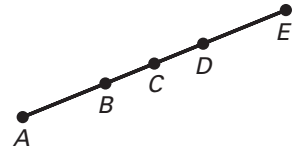
Given: $AC = CE$, $AB = DE$

Prove: C is the midpoint of $\overline{BD}$.

Statements	Reasons
1. $AC = CE$	1. Given
2. $AB + BC = AC$	2. Segment Addition Postulate
3. $AB + BC = CE$	3. Transitive Property of Equality
4. $CD + DE = CE$	4. Segment Addition Postulate
5. $AB + BC = CD + DE$	5. Transitive Property of Equality
6. $AB = DE$	6. Given
7. $AB + BC = CD + AB$	7. Substitution Property of Equality
8. $BC = CD$	8. Subtraction Property of Equality
9. $\overline{BC} \cong \overline{CD}$	9. Definition of congruent segments
10. C is the midpoint of $\overline{BD}$.	10. Definition of midpoint

Checkpoint **Complete the following exercise.**

1. In the diagram, $AB = DE$ and $BC = CD$. Complete the proof to show that C is the midpoint of $\overline{AE}$.



Given: $AB = DE, BC = CD$

Prove: C is the midpoint of $\overline{AE}$.

Statements	Reasons
1. $AB = DE$	**1.** Given
2. $AB + BC = DE + BC$	**2.** Addition Property of Equality
3. $BC = CD$	**3.** Given
4. $AB + BC = DE + CD$	**4.** Substitution Property of Equality
5. $AB + BC = AC$	**5.** Segment Addition Postulate
6. $AC = DE + CD$	**6.** Transitive Property of Equality
7. $CD + DE = CE$	**7.** Segment Addition Postulate
8. $AC = CE$	**8.** Transitive Property of Equality
9. $\overline{AC} \cong \overline{CE}$	**9.** Definition of congruent segments
10. C is the midpoint of $\overline{AE}$.	**10.** Definition of midpoint