

# 2.6

## Proving Statements about Angles

- Goals**
- Use angle congruence properties.
  - Prove properties about special pairs of angles.

### THEOREM 2.2 PROPERTIES OF ANGLE CONGRUENCE

Angle congruence is reflexive, symmetric, and transitive.

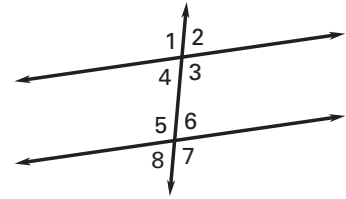
**Reflexive** For any angle  $A$ ,  $\angle A \cong \angle A$ .

**Symmetric** If  $\angle A \cong \angle B$ , then  $\angle B \cong \angle A$ .

**Transitive** If  $\angle A \cong \angle B$  and  $\angle B \cong \angle C$ , then  $\angle A \cong \angle C$ .

### Example 1 Using the Transitive Property

In the diagram at the right,  $\angle 1 \cong \angle 5$ ,  $\angle 5 \cong \angle 3$ , and  $m\angle 1 = 103^\circ$ . What is the measure of  $\angle 3$ ? Explain your reasoning.



#### Solution

Because  $\angle 1 \cong \angle 5$  and  $\angle 5 \cong \angle 3$ , you can use the **Transitive Property of Congruence** to conclude that  $\angle 1 \cong \angle 3$ .

Because congruent angles have the same measure, you can conclude that  $m\angle 3 = m\angle 1 = 103^\circ$ .

**Answer** The measure of  $\angle 3$  is  $103^\circ$ .

### ✓ Checkpoint Use the diagram from Example 1.

1. Given that  $\angle 4 \cong \angle 6$ ,  $\angle 6 \cong \angle 8$ , and  $m\angle 8 = 77^\circ$ , what is the measure of  $\angle 4$ ? Explain your reasoning.

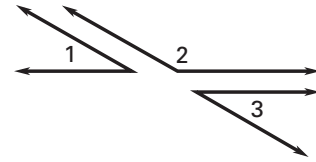
$77^\circ$ ; Using the Transitive Property of Congruence,  $\angle 4 \cong \angle 8$ . Because  $m\angle 8 = 77^\circ$ ,  $m\angle 4 = 77^\circ$ .

### THEOREM 2.3 RIGHT ANGLE CONGRUENCE THEOREM

All right angles are congruent.

### THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM

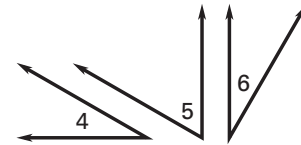
If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.



If  $m\angle 1 + m\angle 2 = 180^\circ$  and  $m\angle 2 + m\angle 3 = 180^\circ$ , then  $\angle 1 \cong \angle 3$ .

### THEOREM 2.5 CONGRUENT COMPLEMENTS THEOREM

If two angles are complementary to the same angle (or to congruent angles), then the two angles are congruent.

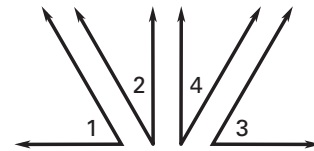


If  $m\angle 4 + m\angle 5 = 90^\circ$  and  $m\angle 5 + m\angle 6 = 90^\circ$ , then  $\angle 4 \cong \angle 6$ .

#### Example 2 Proving Theorem 2.5

Given:  $\angle 1$  and  $\angle 2$  are complements,  
 $\angle 3$  and  $\angle 4$  are complements,  
 $\angle 2 \cong \angle 4$

Prove:  $\angle 1 \cong \angle 3$

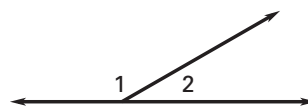


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complements, $\angle 3$ and $\angle 4$ are complements, $\angle 2 \cong \angle 4$	1. <u>Given</u>
2. $m\angle 1 + m\angle 2 = 90^\circ$ , $m\angle 3 + m\angle 4 = 90^\circ$	2. <u>Def. of complementary angles</u>
3. $\underline{m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4}$	3. Transitive property of equality
4. $m\angle 2 = m\angle 4$	4. <u>Def. of congruent angles</u>
5. $\underline{m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2}$	5. Substitution property of equality
6. $\underline{m\angle 1 = m\angle 3}$	6. Subtraction property of equality
7. $\angle 1 \cong \angle 3$	7. <u>Def. of congruent angles</u>

## POSTULATE 12 LINEAR PAIR POSTULATE

If two angles form a linear pair, then they are supplementary.

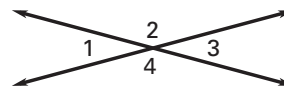
$$m\angle 1 + m\angle 2 = \underline{180^\circ}$$



## THEOREM 2.6 VERTICAL ANGLES THEOREM

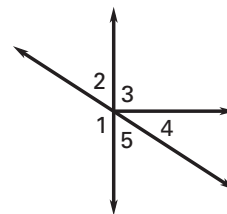
Vertical angles are congruent.

$$\angle 1 \cong \underline{\angle 3} \text{ and } \underline{\angle 2} \cong \angle 4$$



### Example 3 Using Linear Pairs and Vertical Angles

In the diagram,  $\angle 3$  is a right angle and  $m\angle 5 = 57^\circ$ . Find the measures of  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ , and  $\angle 4$ .



#### Solution

By the definition of a right angle,  $m\angle 3 = \underline{90^\circ}$ .

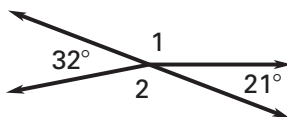
$\angle 2$  and  $\angle 5$  are vertical angles and  $m\angle 5 = 57^\circ$ , so  $m\angle 2 = \underline{57^\circ}$ .

$\angle 1$  and  $\angle 5$  form a linear pair, so  $m\angle 1 + m\angle 5 = \underline{180^\circ}$ . When you substitute  $\underline{57^\circ}$  for  $m\angle 5$  and solve for  $m\angle 1$ , the result is  $m\angle 1 = \underline{123^\circ}$ .

$\angle 4$  and  $\angle 5$  are complementary, so  $m\angle 4 + m\angle 5 = \underline{90^\circ}$ . When you substitute  $\underline{57^\circ}$  for  $m\angle 5$  and solve for  $m\angle 4$ , the result is  $m\angle 4 = \underline{33^\circ}$ .

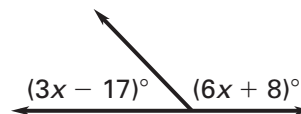
✔ **Checkpoint** Complete the following exercises.

2. Find  $m\angle 1$  and  $m\angle 2$ .



$$m\angle 1 = 159^\circ, m\angle 2 = 148^\circ$$

3. Find the measure of each angle.



$$46^\circ, 134^\circ$$