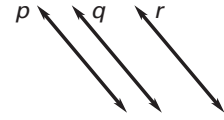# 3.5 Using Properties of Parallel Lines

Goals • Use properties of parallel lines in real-life situations.
• Construct parallel lines using a straightedge and a compass.
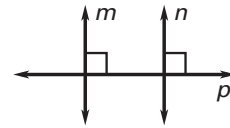

THEOREM 3.11

If two lines are parallel to the same line, then they are _parallel_ to each other.

If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

THEOREM 3.12

In a plane, if two lines are perpendicular to the same line, then they are _parallel_ to each other.
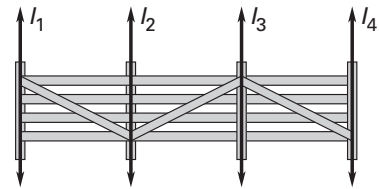
If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Example 1 *Explaining Why Fence Posts are Parallel*

In the diagram at the right, each fence post is parallel to the fence post immediately to the right. Explain why the fence posts on each end are parallel.
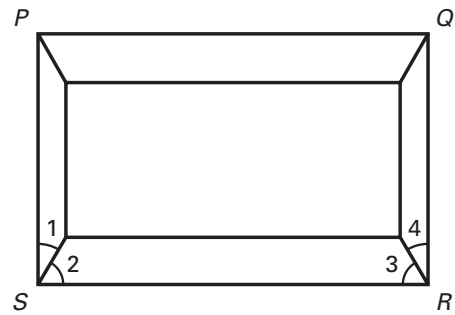
Solution

You are given that $l_1 \parallel$ _l_2_ and $l_2 \parallel$ _l_3_ . By Theorem 3.11, _l_1_ $\parallel$ _l_3_ . You are also given that $l_3 \parallel$ _l_4_ . Because _l_1_ $\parallel$ _l_3_ and _l_3_ $\parallel$ _l_4_ , you can use Theorem 3.11 again to conclude that $l_1 \parallel l_4$.

Example 2   *Building a Picture Frame*

You are building the picture frame in the diagram at the right. You cut pieces of wood such that the measures of ∠1 and ∠4 are 30° and the measures of ∠2 and ∠3 are 60°. Prove that the right and left sides of the frame are parallel.

Solution

Given: $m\angle 1 =$ ⎯30⎯ °; $m\angle 2 =$ ⎯60⎯ °; $m\angle 3 =$ ⎯60⎯ °; $m\angle 4 =$ ⎯30⎯ °

Prove: ⎯PS⎯ ∥ ⎯QR⎯

Statements	Reasons
1. $m\angle 1 =$ ⎯30⎯ °, $m\angle 2 =$ ⎯60⎯ ° $m\angle 3 =$ ⎯60⎯ °, $m\angle 4 =$ ⎯30⎯ °	1. ⎯Given⎯
2. $m\angle PSR = m\angle$ ⎯1⎯ $+ m\angle$ ⎯2⎯ $m\angle QRS = m\angle$ ⎯3⎯ $+ m\angle$ ⎯4⎯	2. Angle Addition Postulate
3. $m\angle PSR =$ ⎯90⎯ ° $m\angle QRS =$ ⎯90⎯ °	3. Substitution property
4. ∠PSR is a ⎯right angle⎯ . ∠QRS is a ⎯right angle⎯ .	4. ⎯Definition of right angle⎯
5. $\overline{PS} \perp \overline{SR}$ $\overline{QR} \perp \overline{RS}$	5. ⎯Definition of ⊥ lines⎯
6. ⎯PS⎯ ∥ ⎯QR⎯	6. ⎯In a plane, 2 lines ⊥ to⎯ ⎯the same line are ∥⎯ .

✔ *Checkpoint* **Complete the following exercise.**

1. Are all the vertical lines parallel? Explain.

Yes; Use Theorem 3.12. Because all the vertical lines are coplanar and perpendicular to the horizontal line, they are parallel to each other.
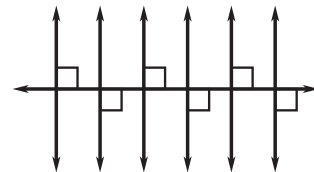
ACTIVITY: CONSTRUCTING PARALLEL LINES

Use the following steps to draw a line that passes through a given point *P* and is parallel to a given line *m*.

1. Draw points *Q* and *R* on line *m*.

2. Draw $\overleftrightarrow{PQ}$.

3. Draw an arc with the compass point at *Q* so that it crosses $\overleftrightarrow{PQ}$ and $\overleftrightarrow{QR}$.

4. Using the same radius, draw an arc with center *P*. Label the intersection of the arc and $\overleftrightarrow{PQ}$ as *T*.

5. Using the point where the initial arc intersects $\overleftrightarrow{PQ}$ as the center, draw an arc that passes through the intersection of the initial arc and $\overleftrightarrow{QR}$.

6. Using the same radius, draw an arc with center *T* that intersects the arc through *T*. Label the intersection as *S*. (Point *S* must be in the interior of ∠*PQR*.)

7. Draw $\overleftrightarrow{PS}$.