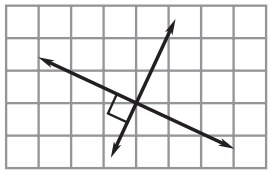
3.7 Perpendicular Lines in the Coordinate Plane

Goals
- Use slope to identify perpendicular lines in coordinate planes.
- Write equations of perpendicular lines.

POSTULATE 18: SLOPES OF PERPENDICULAR LINES

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

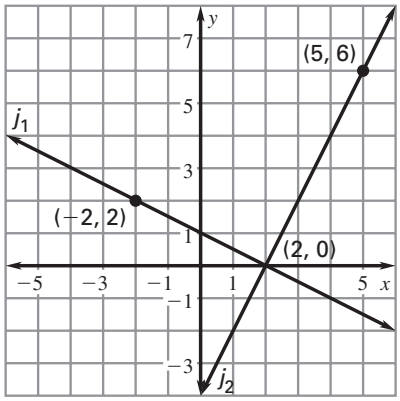
Vertical and horizontal lines are perpendicular.



product of slopes $= 2\left(-\frac{1}{2}\right) = -1$

Example 1 *Deciding Whether Lines are Perpendicular*

Decide whether $j_1 \perp j_2$.



Solution

Find the slope of each line.

Slope of $j_1 = \frac{2 - 0}{-2 - 2} = \frac{2}{-4} = -\frac{1}{2}$

Slope of $j_2 = \frac{6 - 0}{5 - 2} = \frac{6}{3} = 2$

Multiply the slopes.

Answer The product is $\left(-\frac{1}{2}\right)(2) = -1$, so $j_1 \perp j_2$.

Example 2 *Deciding Whether Lines are Perpendicular*

Decide whether the lines are perpendicular.

line *s*: $3x - 2y = 1$ **line *t*:** $6x + 9y = 3$

Solution

Rewrite each equation in slope-intercept form to find the slope.

line *s*: $3x - 2y = 1$

$-2y = \underline{-3x} + 1$

$y = \underline{\frac{3}{2}x - \frac{1}{2}}$

slope $= \underline{\frac{3}{2}}$

line *t*: $6x + 9y = 3$

$9y = \underline{-6x} + 3$

$y = \underline{-\frac{6}{9}x + \frac{3}{9}}$

$y = \underline{-\frac{2}{3}x + \frac{1}{3}}$

slope $= \underline{-\frac{2}{3}}$

The slope-intercept form of a linear equation is $y = mx + b$ where m is the slope and b is the y-intercept.

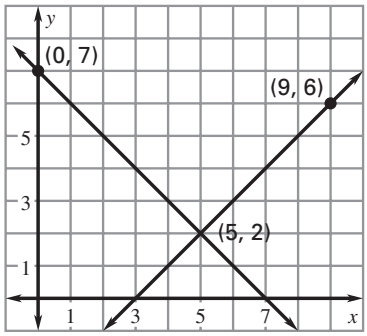
Multiply the slopes to see if the lines are perpendicular.

The product of the slopes is $\underline{-1}$.

Answer So, lines *s* and *t* are $\underline{\text{perpendicular}}$.

✔ ***Checkpoint*** **Find the slopes of the lines. Then decide whether the lines are perpendicular.**

1.



1, −1; yes

2. line k_1: $6x + 2y = 8$

line k_2: $y = -3x - 4$

−3, −3; no

Example 3 *Writing the Equation of a Perpendicular Line*

Line r_1 has equation $y = 3x + 5$. Find an equation of the line r_2 that passes through $P(3, 1)$ and is perpendicular to r_1.

Solution

Find the slope of r_2. Let m_1 and m_2 represent the slopes of r_1 and r_2.

$m_1 \cdot m_2 = \underline{-1}$ **The product of the slopes of $\perp$ lines is $\underline{-1}$.**

$\underline{3} \cdot m_2 = \underline{-1}$ **Substitute for m_1.**

$m_2 = \underline{-\frac{1}{3}}$ **Solve for m_2.**

Then use $m_2 = \underline{-\frac{1}{3}}$ and $(x, y) = (\underline{3}, \underline{1})$ to find b.

$y = m_2x + b$ **Slope-intercept form**

$\underline{1} = \underline{-\frac{1}{3}}(\underline{3}) + b$ **Substitute for y, m_2, and x.**

$\underline{2} = b$ **Simplify.**

Answer So, an equation of r_2 is $y = \underline{-\frac{1}{3}x + 2}$.

✔ ***Checkpoint*** **Find an equation of the line that passes through the given point and is perpendicular to the given line.**

3. $(0, -4)$, $y = -x$ $y = x - 4$	**4.** $(2, -2)$, $y = \frac{1}{4}x + 10$ $y = -4x + 6$