

4.2

Congruence and Triangles

- Goals**
- Identify congruent figures and corresponding parts.
 - Prove that two triangles are congruent.

VOCABULARY

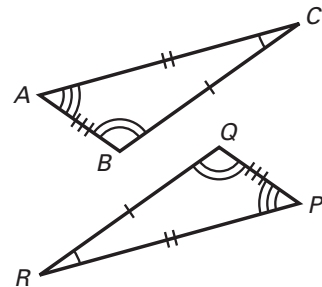
Congruent Two geometric figures are congruent if they have exactly the same size and shape.

Corresponding angles When two figures are congruent, the corresponding angles are the angles that are in corresponding positions and are congruent.

Corresponding sides When two figures are congruent, the corresponding sides are the sides that are in corresponding positions and are congruent.

Example 1 Naming Congruent Parts

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle ABC \cong \triangle PQR$.

The congruent angles and sides are as follows.

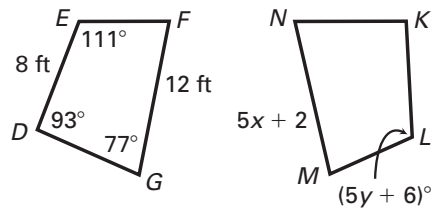
Angles: $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

Sides: $\overline{AB} \cong \overline{PQ}$, $\overline{BC} \cong \overline{QR}$, $\overline{CA} \cong \overline{RP}$

Example 2 Using Properties of Congruent Figures

In the diagram, $DEFG \cong KLMN$.

- Find the value of x .
- Find the value of y .



Solution

- You know that $\overline{FG} \cong \overline{MN}$.

So, $FG = MN$.

$$12 = 5x + 2 \quad \text{Substitute for } \underline{FG} \text{ and } \underline{MN}.$$

$$\underline{10} = 5x \quad \text{Subtract } \underline{2} \text{ from each side.}$$

$$\underline{2} = x \quad \text{Divide each side by } \underline{5}.$$

- You know that $\angle E \cong \angle L$.

So, $m\angle E = m\angle L$.

$$111^\circ = (5y + 6)^\circ \quad \text{Substitute for } \underline{m\angle E} \text{ and } \underline{m\angle L}.$$

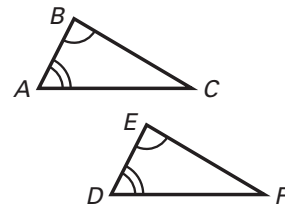
$$\underline{105} = 5y \quad \text{Subtract } \underline{6} \text{ from each side.}$$

$$\underline{21} = y \quad \text{Divide each side by } \underline{5}.$$

THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,
then $\underline{\angle C \cong \angle F}$.



Example 3 Using the Third Angles Theorem

Find the value of x .

Solution

In the diagram, $\angle V \cong \angle Y$ and $\angle U \cong \angle Z$. From the Third Angles Theorem, you know that $\angle W \cong \angle X$.

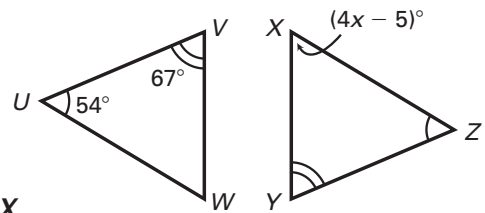
So, $m\angle W = m\angle X$. From the Triangle Sum Theorem, $m\angle W = 180^\circ - 54^\circ - 67^\circ = \underline{59^\circ}$.

$$m\angle W = m\angle X \quad \text{Third Angles Theorem}$$

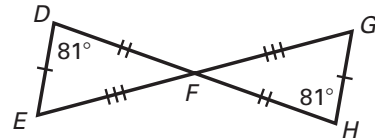
$$\underline{59^\circ} = (4x - 5)^\circ \quad \text{Substitute.}$$

$$\underline{64} = 4x \quad \text{Add } \underline{5} \text{ to each side.}$$

$$\underline{16} = x \quad \text{Divide each side by } \underline{4}.$$

**Example 4** Determining Whether Triangles are Congruent

Decide whether the triangles are congruent. Justify your reasoning.

**Solution**

Paragraph Proof From the diagram, you are given that all three pairs of corresponding sides are congruent.

$$\overline{DE} \cong \overline{HG}, \overline{EF} \cong \overline{GF}, \overline{DF} \cong \overline{HF}$$

Because $\angle D$ and $\angle H$ have the same measure, $\angle D \cong \angle H$. By the Vertical Angles Theorem, you know that $\underline{\angle DFE \cong \angle HFG}$.

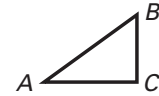
By the Third Angles Theorem, $\underline{\angle E \cong \angle G}$.

Answer So, all three pairs of corresponding sides and all three pairs of corresponding angles are congruent. By the definition of congruent triangles, $\underline{\triangle DEF \cong \triangle HGF}$.

THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES

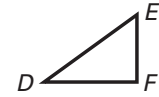
Reflexive Property of Congruent Triangles

Every triangle is congruent to itself.



Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

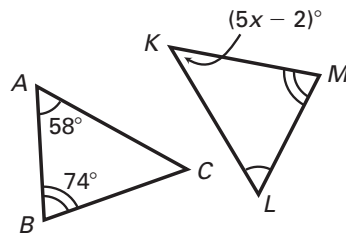
If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



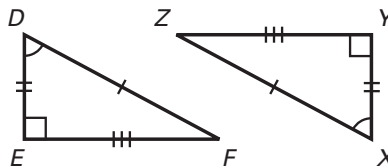
✔ **Checkpoint** Complete the following exercises.

1. Find the value of x .

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2. Decide whether the triangles are congruent. Justify your reasoning.



From the diagram, it is given that $\overline{DE} \cong \overline{XY}$, $\overline{EF} \cong \overline{YZ}$, $\overline{DF} \cong \overline{XZ}$, $\angle D \cong \angle X$, and $\angle E \cong \angle Y$. By the Third Angles Theorem, $\angle DFE \cong \angle XZY$. By the definition of congruent triangles, $\triangle DEF \cong \triangle XYZ$.