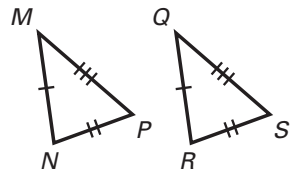
4.3 Proving Triangles are Congruent: SSS and SAS

Goals
- Prove that triangles are congruent using the SSS and SAS Congruence Postulates.
- Use congruence postulates in real-life problems.

POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE

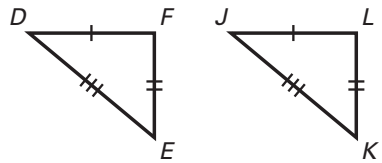
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{MN} \cong$ $\overline{QR}$,
Side $\overline{NP} \cong$ $\overline{RS}$, and
Side $\overline{PM} \cong$ $\overline{SQ}$,
then $\triangle MNP \cong$ $\triangle QRS$.



Example 1 *Using the SSS Congruence Postulate*

Prove that $\triangle DEF \cong \triangle JKL$.



Solution

Paragraph Proof The marks on the diagram show that $\overline{DE} \cong$ $\overline{JK}$, $\overline{EF}$ $\cong \overline{KL}$, and $\overline{DF} \cong$ $\overline{JL}$.

Answer So, by the SSS Congruence Postulate, you know that $\triangle DEF$ $\cong$ $\triangle JKL$.

POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE

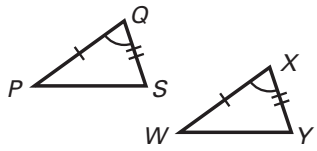
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{PQ} \cong$ $\overline{WX}$,

Angle $\angle Q \cong$ $\angle X$, and

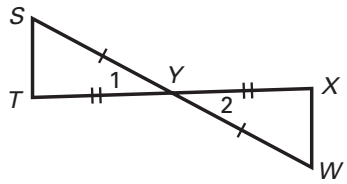
Side $\overline{QS} \cong$ $\overline{XY}$,

then $\triangle PQS \cong$ $\triangle WXY$.



Example 2 ***Using the SAS Congruence Postulate***

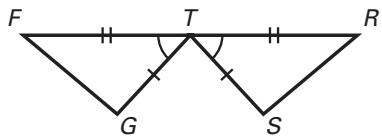
Prove that $\triangle SYT \cong \triangle WYX$.



Statements	Reasons
1. $\overline{TY} \cong$ $\overline{XY}$, $\overline{SY} \cong$ $\overline{WY}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Theorem
3. $\triangle SYT \cong \triangle WYX$	3. SAS Congruence Postulate

✔ ***Checkpoint*** **Complete the following exercise.**

1. Prove that $\triangle FGT \cong \triangle RST$.

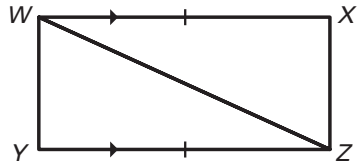


Statements (Reasons)

1. $\overline{GT} \cong \overline{ST}$, $\overline{FT} \cong \overline{RT}$ (Given)
2. $\angle FTG \cong \angle RTS$ (Given)
3. $\triangle FGT \cong \triangle RST$ (SAS)

Example 3 ***Choosing a Congruence Postulate to Use***

Decide whether enough information is given in the diagram to prove that $\triangle WYZ \cong \triangle ZXW$. If there is enough information, state the congruence postulate you would use.

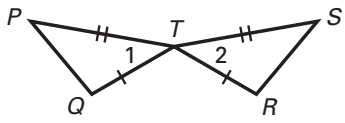


Solution

Paragraph Proof The marks on the diagram show that $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{YZ}$. By the Alternate Interior Angles Postulate, $\angle WZY \cong \angle ZWX$. By the Reflexive Property of Congruence, $\overline{WZ} \cong \overline{WZ}$. Because two sides and the included angle of $\triangle WYZ$ are congruent to the corresponding two sides and included angle of $\triangle ZXW$, you can use the SAS Congruence Postulate to prove that the triangles are congruent.

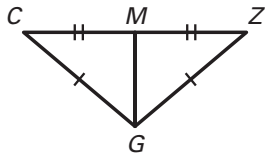
✔ ***Checkpoint*** **Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate you would use.**

2. $\triangle PTQ \cong \triangle STR$



yes; SAS Congruence Postulate

3. $\triangle CMG \cong \triangle ZMG$



yes; SSS Congruence Postulate