

4.4

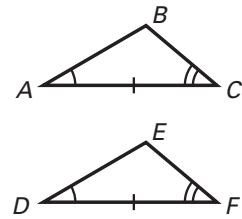
Proving Triangles are Congruent: ASA and AAS

- Goals**
- Prove that triangles are congruent using the ASA Congruence Postulate and the AAS Congruence Theorem.
 - Use congruence postulates and theorems in real-life problems.

POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

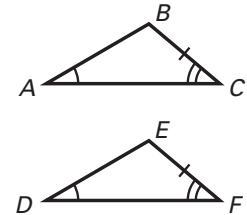
If Angle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.5: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

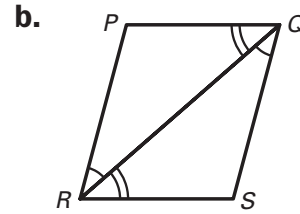
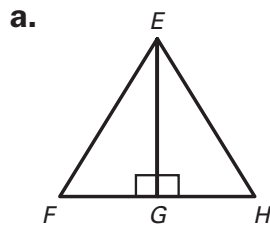
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of a second triangle, then the two triangles are congruent.

If Angle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Example 1 *Developing Proof*

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.



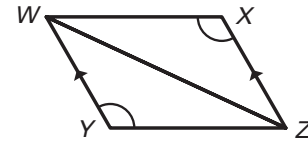
Solution

- a. The diagram shows that $\angle EGH$ and $\angle EGF$ are right angles. So, $\angle EGH \cong \angle EGF$. Also, $\overline{EG} \cong \overline{EG}$ by the Reflexive Property of Congruence. This is not enough information to prove that the triangles are congruent.
- b. In addition to the angles that are marked, $\overline{QR} \cong \overline{QR}$ by the Reflexive Property of Congruence. Two pairs of corresponding angles and the one pair of corresponding sides are congruent. You can use the ASA Congruence Postulate to prove that $\triangle PQR \cong \triangle SRQ$.

Example 2 *Proving Triangles are Congruent*

Given: $\overline{WY} \parallel \overline{XZ}$, $\angle Y \cong \angle X$

Prove: $\triangle WYZ \cong \triangle ZXW$

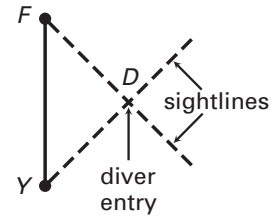


Plan for Proof You are given that $\angle Y \cong \angle X$. Use the fact that $\overline{WY} \parallel \overline{XZ}$ to identify a pair of congruent angles.

Statements	Reasons
1. $\angle Y \cong \angle X$, $\overline{WY} \parallel \overline{XZ}$	1. Given
2. $\angle YWZ \cong \angle XZW$	2. Alternate Interior Angles Theorem
3. $\overline{WZ} \cong \overline{WZ}$	3. Reflexive Property of Congruence
4. $\triangle WYZ \cong \triangle ZXW$	4. <u>AAS Congruence Theorem</u>

Example 3 Using Properties of Congruent Triangles

Cliff Diving At a cliff-diving competition, you and a friend stand at different locations (points F and Y) along the shore. To record the spot where a diver enters the water, you find the angle between each of your sightlines and \overline{FY} . Assuming your sightlines are accurate, do you have enough information to record a diver's entry data?

**Solution**

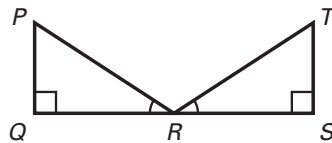
Think of points F and Y as two vertices of a triangle. The diver's entry spot D is the other vertex. You know $m\angle F$ and $m\angle Y$. You also know the length of the included side \overline{FY} . From the **ASA Congruence Postulate**, you can conclude that any two triangles with these measurements are **congruent**. In other words, there is only one triangle with the given measurements and location.

Answer You have enough information to record a diver's entry data.

✔ **Checkpoint** Complete the following exercises.

1. **Given:** R is the midpoint of \overline{QS} ; $\angle QRP \cong \angle SRT$; $\angle Q$ and $\angle S$ are right angles.

Prove: $\triangle QRP \cong \triangle SRT$



- Statements (Reasons)**
1. $\angle QRP \cong \angle SRT$ (Given)
 2. $\angle Q \cong \angle S$
(Rt. \angle Congruence Theorem)
 3. R is the midpoint of \overline{QS} .
(Given)
 4. $\overline{QR} \cong \overline{RS}$ (Def. of midpoint)
 5. $\triangle QRP \cong \triangle SRT$
(ASA Congruence Postulate)

2. A kite falls into a field $E 35^\circ N$ from where you stand. Your friend, standing 75 yards from you, did not see the kite fall. Is there enough information to locate the kite after it fell?

No; You only know the distance between you and your friend, and the angle between you and the kite. This is not enough information to know where the kite landed.