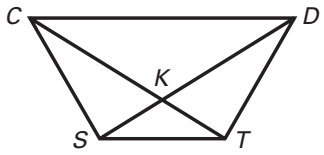
4.5 Using Congruent Triangles

Goals
- Use congruent triangles to plan and write proofs.
- Use congruent triangles to prove constructions are valid.

Example 1 *Planning and Writing a Proof*

Given: $\overline{SD} \cong \overline{TC}$, $\overline{CS} \cong \overline{DT}$

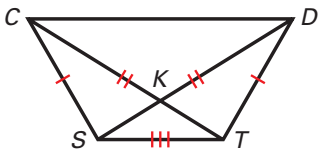
Prove: $\angle SCT \cong \angle TDS$



Plan for Proof Show that $\triangle TDS \cong \triangle SCT$. Then use the fact that corresponding parts of congruent triangles are congruent.

Solution

Mark the diagram at the right with the given information. Then mark any additional information that you can deduce. Because $\overline{ST}$ is the same segment in both triangles, you can deduce that $\overline{ST} \cong \overline{ST}$.

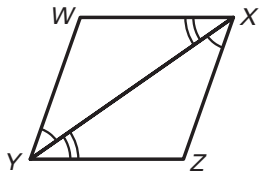


Paragraph Proof By the Reflexive Property of Congruence, $\overline{ST}$ $\cong$ $\overline{ST}$. You can use the SSS Congruence Postulate to conclude that $\triangle TDS$ $\cong$ $\triangle SCT$. Finally, because corresponding parts of congruent triangles are congruent, it follows that $\angle SCT$ $\cong$ $\angle TDS$.

✔ ***Checkpoint*** **Complete the following exercise.**

1. Given: $\angle WYX \cong \angle ZXY$, $\angle WXY \cong \angle ZYX$

Prove: $\overline{WX} \cong \overline{ZY}$

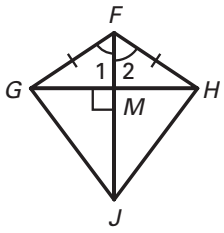


By the Reflexive Property of Congruence, $\overline{XY} \cong \overline{XY}$. You can use the ASA Congruence Postulate to conclude that $\triangle WYX \cong \triangle ZXY$. Because corresponding parts of congruent triangles are congruent, it follows that $\overline{WX} \cong \overline{ZY}$.

Example 2 *Using More than One Pair of Triangles*

Given: $\angle GMJ$ and $\angle HMJ$ are right angles, $\overline{GF} \cong \overline{HF}$, $\angle 1 \cong \angle 2$

Prove: $\angle GJM \cong \angle HJM$



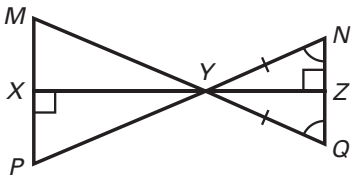
Plan for Proof Prove that $\triangle GJM \cong \triangle HJM$. Then use the fact that corresponding parts of congruent triangles are congruent to show that $\angle GJM \cong \angle HJM$.

Statements	Reasons
1. $\overline{GF} \cong \overline{HF}$, $\angle 1 \cong \angle 2$	1. Given
2. $\overline{FM} \cong \overline{FM}$	2. Reflexive Prop. of Cong.
3. $\triangle FGM \cong \triangle FHM$	3. SAS Congruence Postulate
4. $\overline{GM} \cong \overline{MH}$	4. Corres. parts of $\cong$ $\triangle$s are $\cong$.
5. $\angle GMJ \cong \angle HMJ$	5. Right angles are congruent.
6. $\overline{MJ} \cong \overline{MJ}$	6. Reflexive Prop. of Cong.
7. $\triangle GJM \cong \triangle HJM$	7. SAS Congruence Postulate
8. $\angle GJM \cong \angle HJM$	8. Corres. parts of $\cong$ $\triangle$s are $\cong$.

✔ ***Checkpoint*** **Complete the following exercise.**

2. Given: $\angle NZY$, $\angle QZY$, $\angle MXY$, $\angle PXY$ are right angles, $\overline{YN} \cong \overline{YQ}$, $\angle N \cong \angle Q$

Prove: $\triangle MYX \cong \triangle PYX$

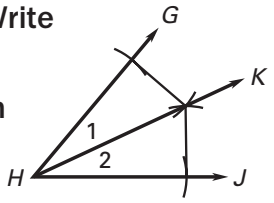


Statements (Reasons)

1. $\overline{YN} \cong \overline{YQ}$, $\angle N \cong \angle Q$ (Given)
2. $\angle NZY \cong \angle QZY$ (Right angles are congruent.)
3. $\triangle NYZ \cong \triangle QYZ$ (AAS Congruence Theorem)
4. $\angle NYZ \cong \angle QYZ$ (Corres. parts of $\cong$ $\triangle$s are $\cong$.)
5. $\angle MYX \cong \angle QYZ$, $\angle PYX \cong \angle NYZ$ (Vertical Angles Theorem)
6. $\angle MYX \cong \angle PYX$ (Transitive Property of Angle Congruence)
7. $\overline{XY} \cong \overline{XY}$ (Reflexive Property of Congruence)
8. $\angle MXY \cong \angle PXY$ (Right angles are congruent.)
9. $\triangle MYX \cong \triangle PYX$ (ASA Congruence Postulate)

Example 3 ***Proving a Construction***

The construction shows $\angle GHJ$ bisected by $\overrightarrow{HK}$. Write a proof to verify that the construction is valid.



Plan for Proof Show that $\triangle HGK \cong \triangle HJK$. Then show that $\angle 1 \cong \angle 2$. By construction, you can assume the following statements as given.

$\overline{HG} \cong \overline{HJ}$ **Same compass setting is used.**

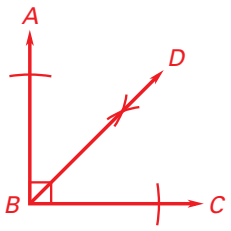
$\overline{GK} \cong \overline{JK}$ **Same compass setting is used.**

Solution

Statements	Reasons
1. $\overline{HG} \cong \overline{HJ}$	1. Given
2. $\overline{GK} \cong \overline{JK}$	2. Given
3. $\overline{HK} \cong \overline{HK}$	3. Reflexive Property of Congruence
4. $\triangle HGK \cong \triangle HJK$	4. SSS Congruence Postulate
5. $\angle 1 \cong \angle 2$	5. Corresp. parts of $\cong \triangle$s are $\cong$.
6. $\overrightarrow{HK}$ bisects $\angle GHJ$	6. Definition of angle bisector

Checkpoint **Use a straightedge and a compass to perform the construction. Label the important points of your construction. Then write a paragraph proof to verify the results.**

3. Bisect a right angle.



$\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{DC}$ is given. By the Reflexive Property of Congruence, $\overline{BD} \cong \overline{BD}$. $\triangle ABD \cong \triangle CBD$ by the SSS Congruence Postulate. $\angle ABD \cong \angle CBD$ because corresponding parts of congruent triangles are congruent. Then, by definition, $\overrightarrow{BD}$ bisects $\angle B$.