4.6 Isosceles, Equilateral, and Right Triangles

Goals
- Use properties of isosceles and equilateral triangles.
- Use properties of right triangles.

VOCABULARY

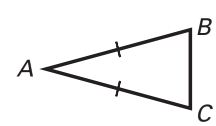
Base angles Base angles are the two angles adjacent to the base of a triangle.

Vertex angle The vertex angle is the angle opposite the base of a triangle.

THEOREM 4.6: BASE ANGLES THEOREM

If two sides of a triangle are congruent, then the angles opposite them are congruent.

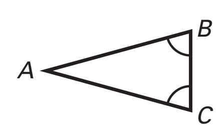
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle$ C.



THEOREM 4.7: CONVERSE OF THE BASE ANGLES THEOREM

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong$ $\overline{AC}$.



COROLLARY TO THEOREM 4.6

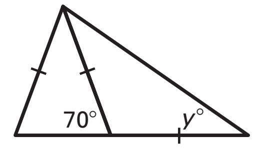
If a triangle is equilateral, then it is equiangular.

COROLLARY TO THEOREM 4.7

If a triangle is equiangular, then it is equilateral.

Example 1 *Using Isosceles Triangles*

Find the value of y.



Solution

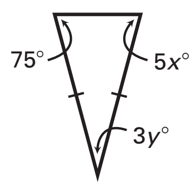
Notice that y represents the measure of a base angle of an isosceles triangle. From the Base Angles Theorem, the other base angle has the same measure. The vertex angle forms a linear pair with a 70° angle, so its measure is 110°.

$110° + 2y° = 180°$ **Apply the Triangle Sum Theorem.**

$y = 35$ **Solve for y.**

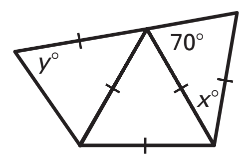
Checkpoint **Solve for x and y.**

1.



$x = 15,\ y = 10$

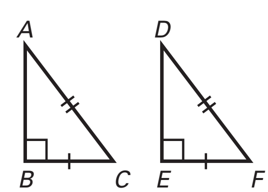
2.



$x = 40,\ y = 65$

THEOREM 4.8: HYPOTENUSE-LEG (HL) CONGRUENCE THEOREM

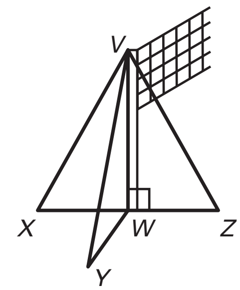
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

Example 2 ***Proving Right Triangles Congruent***

The pole holding up one end of a volleyball net is perpendicular to the plane containing the points *W*, *X*, *Y*, and *Z*. Each of the lines running from the top of the pole to *X*, *Y*, and *Z* uses the same length of rope. Prove that $\triangle VWX$, $\triangle VWY$, and $\triangle VWZ$ are congruent.



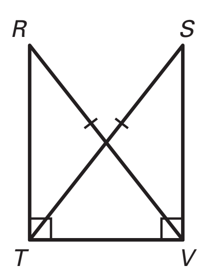
Given: $\overline{VW} \perp \overline{WX}$, $\overline{VW} \perp \overline{WY}$, $\overline{VW} \perp \overline{WZ}$, $\overline{VX} \cong \overline{VY} \cong \overline{VZ}$

Prove: $\triangle VWX \cong \triangle VWY \cong \triangle VWZ$

Paragraph Proof You are given that $\overline{VW} \perp \overline{WX}$ and $\overline{VW} \perp \overline{WY}$, which implies that $\angle VWX$ and $\angle VWY$ are right angles. By definition, $\triangle VWX$ and $\triangle VWY$ are right triangles. You are given that the hypotenuses of these two triangles, $\overline{VX}$ and $\overline{VY}$, are congruent. Also, $\overline{VW}$ is a leg for both triangles, and $\overline{VW} \cong \overline{VW}$ by the Reflexive Property of Congruence. Thus, by the Hypotenuse-Leg Congruence Theorem, $\triangle VWX \cong \triangle VWY$. Similar reasoning can be used to prove that $\triangle VWY \cong \triangle VWZ$. So, by the Transitive Property of Congruent Triangles, $\triangle VWX \cong \triangle VWY \cong \triangle VWZ$.

Checkpoint **Complete the following exercise.**

3. Given: $\overline{RV} \cong \overline{ST}$; $\angle RTV$ and $\angle SVT$ are right angles
Prove: $\triangle RTV \cong \triangle SVT$



Statements (Reasons)
1. $\overline{RV} \cong \overline{ST}$ (Given)
2. $\angle RTV$ and $\angle SVT$ are right angles (Given)
3. $\angle RTV \cong \angle SVT$ (Right angles are congruent.)
4. $\overline{TV} \cong \overline{TV}$ (Reflexive Property of Congruence)
5. $\triangle RTV \cong \triangle SVT$ (HL Congruence Theorem)