4.7 Triangles and Coordinate Proof

Goals
- Place geometric figures in a coordinate plane.
- Write a coordinate proof.

VOCABULARY

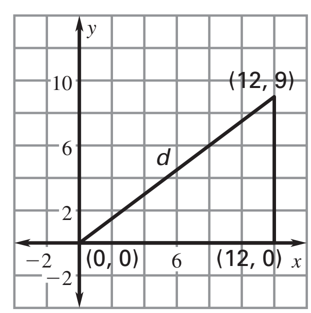
Coordinate proof A coordinate proof is a type of proof that involves placing geometric figures in a coordinate plane.

Example 1 *Using the Distance Formula*

A right triangle has legs of 9 units and 12 units. Place the triangle in a coordinate plane. Label the coordinates of the vertices and find the length of the hypotenuse.

Solution

One possible placement is shown. Notice that one leg is vertical and the other leg is horizontal, which assures that the legs meet at right angles. Points on the same vertical segment have the same x-coordinate, and the points on the same horizontal segment have the same y-coordinate.



You can use the Distance Formula to find the length of the hypotenuse.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Distance Formula**

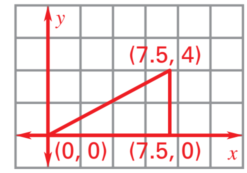
$= \sqrt{(12 - 0)^2 + (9 - 0)^2}$ **Substitute.**

$= \sqrt{225}$ **Simplify.**

$= 15$ **Evaluate square root.**

Checkpoint Compete the following exercise.

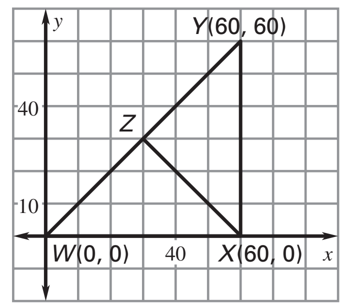
1. A right triangle has legs of 7.5 units and 4 units. Place the triangle in a coordinate plane. Label the vertices and find the length of the hypotenuse.



8.5 units

Example 2 *Using the Midpoint Formula*

In the diagram, $\triangle WXZ \cong \triangle YXZ$. Find the coordinates of point Z.



Solution

Because the triangles are congruent, it follows that $\overline{WZ} \cong$ $\underline{\overline{YZ}}$. So point Z must be the midpoint of $\underline{\overline{WY}}$. This means you can use the __Midpoint Formula__ to find the coordinates of point Z.

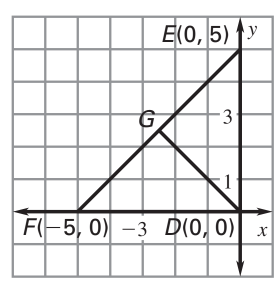
$$Z(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ **Midpoint Formula**

$$= \left(\frac{\boxed{0} + \boxed{60}}{2}, \frac{\boxed{0} + \boxed{60}}{2}\right)$$ **Substitute.**

$$= (\underline{30}, \underline{30})$$ **Simplify.**

Checkpoint Complete the following exercise.

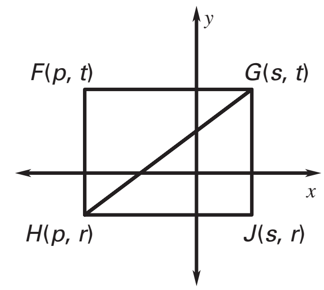
2. In the diagram, $\triangle DEG \cong \triangle DFG$. Find the coordinates of G.



(−2.5, 2.5)

Example 3 *Writing a Coordinate Proof*

Given: Coordinates of figure *FGJH*
Prove: $\triangle FGH \cong \triangle JHG$



Solution

Use the Distance Formula to find *FG* and *HJ*.

$$FG = \sqrt{(s - p)^2 + (t - t)^2} = \sqrt{(s - p)^2}$$

$$HJ = \sqrt{(s - p)^2 + (r - r)^2} = \sqrt{(s - p)^2}$$

Use the Distance Formula to find *FH* and *GJ*.

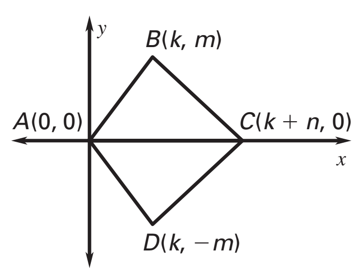
$$FH = \sqrt{(p - p)^2 + (r - t)^2} = \sqrt{(r - t)^2}$$

$$GJ = \sqrt{(s - s)^2 + (r - t)^2} = \sqrt{(r - t)^2}$$

So, you can conclude that $\overline{FG} \cong \overline{HJ}$ and $\overline{FH} \cong \overline{GJ}$. Because $\overline{GH} \cong \overline{GH}$, you can apply the SSS Congruence Postulate to conclude that $\triangle FGH \cong \triangle JHG$.

Checkpoint Complete the following exercise.

3. **Given:** Coordinates of figure *ABCD*
Prove: $\triangle ABC \cong \triangle ADC$



$AB = \sqrt{k^2 + m^2}$; $AD = \sqrt{k^2 + m^2}$; $BC = \sqrt{n^2 + m^2}$; $DC = \sqrt{n^2 + m^2}$; By the Reflexive Property of Congruence, $\overline{AC} \cong \overline{AC}$. So, by the SSS Congruence Postulate, $\triangle ABC \cong \triangle ADC$.