5.1 Perpendiculars and Bisectors

Goals
- Use properties of perpendicular bisectors.
- Use properties of angle bisectors to identify equal distances.

VOCABULARY

Perpendicular bisector A perpendicular bisector is a segment, ray, line, or plane that is perpendicular to a segment at its midpoint.

Equidistant from two points A point is equidistant from two points if its distance from each point is the same.

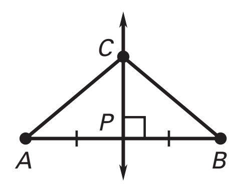
Distance from a point to a line The distance from a point to a line is the length of the perpendicular segment from the point to the line.

Equidistant from two lines A point is equidistant from two lines when the point is the same distance from one line as it is from another line.

THEOREM 5.1: PERPENDICULAR BISECTOR THEOREM

If a point is on the perpendicular bisector of a segment, then it is equidistant from the _endpoints_ of the segment.

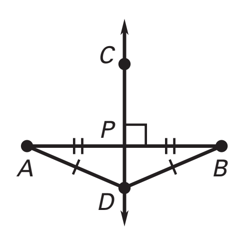
If $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$, then _CA_ = _CB_.



THEOREM 5.2: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

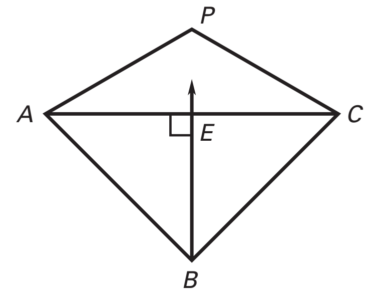
If a point is _equidistant_ from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If _DA_ = _DB_, then D lies on the perpendicular bisector of $\overline{AB}$.



Example 1 *Using Perpendicular Bisectors*

In the diagram shown, $\overrightarrow{BE}$ is the perpendicular bisector of $\overline{AC}$.



a. What segment lengths are equal?

b. $\overline{AP} \cong \overline{CP}$. What can you conclude about point *P*?

Solution

a. Because $\overrightarrow{BE}$ bisects $\overline{AC}$, ___AE___ = ___CE___.

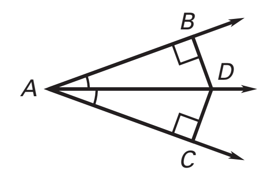
Because *B* is on the perpendicular bisector of $\overline{AC}$, you can use the ___Perpendicular Bisector___ Theorem to conclude that ___AB___ = ___BC___.

b. Because $\overline{AP} \cong \overline{CP}$, $AP =$ ___CP___. Using the ___Converse of the Perpendicular Bisector___ Theorem, you can conclude that ___P lies on $\overrightarrow{BE}$___.

THEOREM 5.3: ANGLE BISECTOR THEOREM

If a point is on the bisector of an angle, then it is equidistant from the two ___sides___ of the angle.

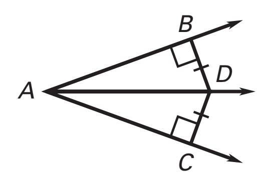
If $m\angle$ ___BAD___ $= m\angle$ ___CAD___, then $DB = DC$.



THEOREM 5.4: CONVERSE OF THE ANGLE BISECTOR THEOREM

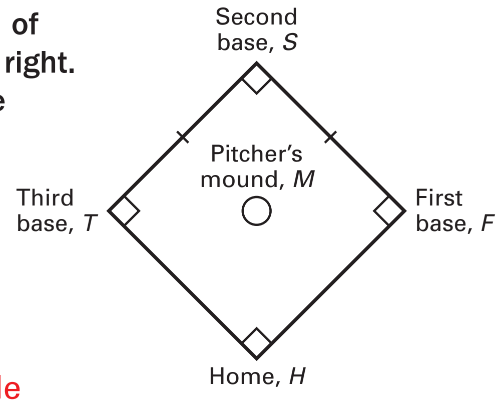
If a point is in the interior of an angle and is equidistant from the ___sides___ of the angle, then it lies on the bisector of the angle.

If ___DB___ = ___DC___, then $m\angle BAD = m\angle CAD$.



Example 2 ***Using Angle Bisectors***

Baseball Field Use the diagram of the baseball infield shown at the right. What can you conclude about the measure of $\angle SHF$?



Solution

From the diagram, you know that point S is in the interior of $\angle THF$ and ST = SF.

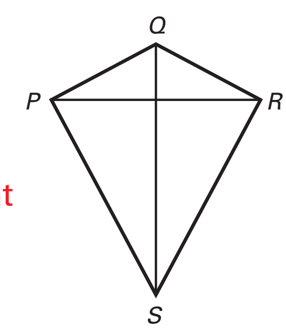
From the Converse of the Angle Bisector Theorem, you know that S lies on the angle bisector of ∠THF. An angle bisector divides an angle into two congruent angles, each of which has half the measure of the original angle, so

$$m\angle SHF = \frac{90^\circ}{2} = 45^\circ.$$

Answer The measure of $\angle SHF$ is 45°.

Checkpoint Complete the following exercises.

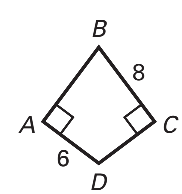
1. In the diagram, $\overline{PQ} \cong \overline{RQ}$. What conclusion can you make about point Q? Can you conclude that S is on the perpendicular bisector of $\overline{PR}$? Explain.



From Theorem 5.2, you can conclude that Q is on the ⊥ bisector of $\overline{PR}$.

No; you do not know if $PS = RS$, so you cannot conclude that S is also on the perpendicular bisector of $\overline{PR}$.

2. In the diagram, D is on the bisector of $\angle ABC$. What is DC? Explain.



6; From Theorem 5.3, you know that $AD = DC$.