5.2 Bisectors of a Triangle

Goals • Use properties of perpendicular bisectors of a triangle.
• Use properties of angle bisectors of a triangle.

VOCABULARY

Perpendicular bisector of a triangle A perpendicular bisector of a triangle is a line, ray, or segment that is perpendicular to a side of the triangle at the midpoint of the side.

Concurrent lines Concurrent lines are three or more lines that intersect at the same point.

Point of concurrency A point of concurrency is the point of intersection of concurrent lines.

Circumcenter of a triangle A circumcenter of a triangle is the point of concurrency of the perpendicular bisectors of the triangle.

Angle bisector of a triangle An angle bisector of a triangle is a bisector of an angle of the triangle.

Incenter of a triangle An incenter of a triangle is the point of concurrency of the angle bisectors of a triangle.

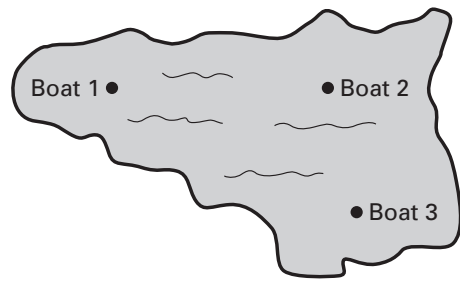**THEOREM 5.5: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE**

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

PA = PB = PC
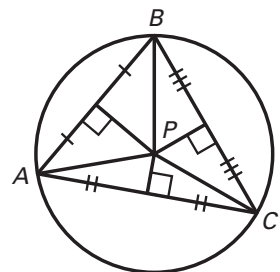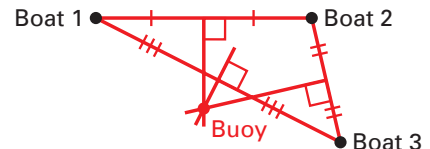
Example 1 *Using Perpendicular Bisectors*

Boating The positions of three boats are shown in the diagram. The boats are equidistant from a buoy. Where is the buoy?

Solution

To find the location of the buoy, find the _circumcenter_ of the triangle formed by the boats' locations.
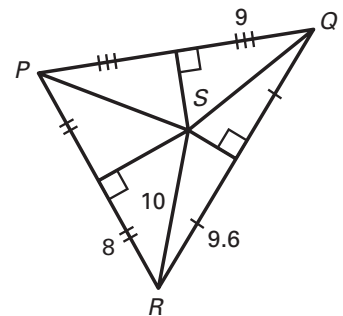
Use the diagram at the right to make a sketch that shows the buoy's location. Label the buoy's location.

✔ *Checkpoint* Complete the following exercise.

1. The perpendicular bisectors of △*PQR* meet at point *S*. Find *PS* and *QS*.
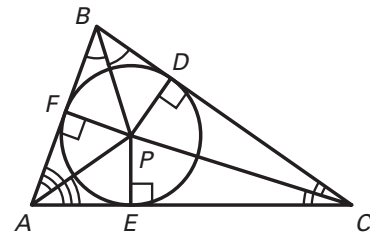
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THEOREM 5.6: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

PD = _PE_ = _PF_

Example 2 *Using Angle Bisectors*

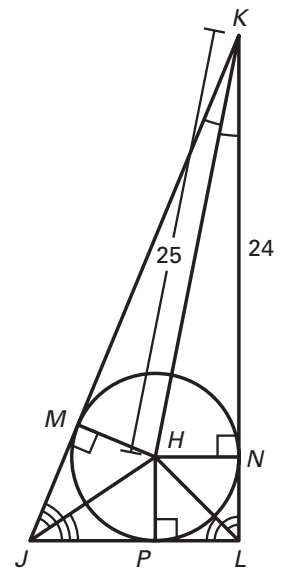In the diagram, the angle bisectors of △*JKL* meet at point *H*.

a. What segments are congruent?

b. Find *HN* and *HP*.

Solution

a. By Theorem 5.6, the three angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle. So, *HM* = *HN* = *HP*. Therefore, $\overline{HM}$, $\overline{HN}$, and $\overline{HP}$ are congruent.

b. Use the Pythagorean Theorem to find *HN* in △*HNK*.

$(HN)^2 + (\underline{\ KN\ })^2 = (\underline{\ HK\ })^2$ **Pythagorean Theorem**

$(HN)^2 + \underline{24}^2 = \underline{25}^2$ **Substitute.**

$(HN)^2 + \underline{576} = \underline{625}$ **Evaluate each square.**

$(HN)^2 = \underline{49}$ **Subtract** $\underline{576}$ **from each side.**

$HN = \underline{7}$ **Find the positive square root.**

Answer So, *HN* = *HP* = $\underline{7}$ units.

✔ *Checkpoint* **Complete the following exercise.**

2. The angle bisectors of △*TUV* meet at point *W*. Find the value of *d*.

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$2d + 7$

$4d - 1$