5.3 Medians and Altitudes of a Triangle

Goals
- Use properties of medians of a triangle.
- Use properties of altitudes of a triangle.

VOCABULARY

Median of a triangle A median of a triangle is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

Centroid of a triangle The centroid of a triangle is the point of concurrency of the medians of the triangle.

Altitude of a triangle An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

Orthocenter of a triangle The orthocenter of a triangle is the point of concurrency of the lines containing the altitudes of the triangle.

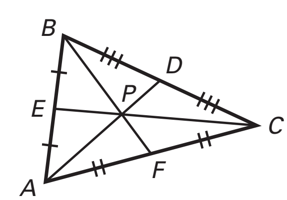
THEOREM 5.7: CONCURRENCY OF MEDIANS OF A TRIANGLE

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

If P is the centroid of $\triangle ABC$, then

$AP = \frac{2}{3}$ AD, $BP = \frac{2}{3}$ BF, and

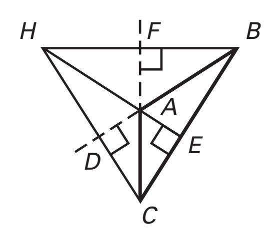
$CP = \frac{2}{3}$ CE.



THEOREM 5.8: CONCURRENCY OF ALTITUDES OF A TRIANGLE

The lines containing the altitudes of a triangle are concurrent.

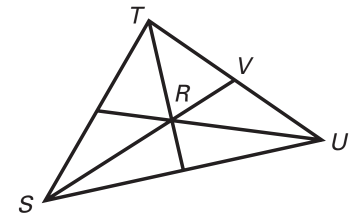
If $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are the altitudes of $\triangle ABC$, then the lines $\overleftrightarrow{AE}$, $\overleftrightarrow{BF}$, and $\overleftrightarrow{CD}$ intersect at some point H.



Example 1 *Using the Centroid of a Triangle*

R is the centroid of $\triangle STU$ and $SR = 16$. Find *SV* and *RV*.

Solution



Because *R* is the centroid, $SR = \underline{\frac{2}{3}}\, SV$.

$\underline{16} = \underline{\frac{2}{3}}\, SV$ **Substitute for *SR*.**

$\underline{24} = SV$ **Simplify.**

By the Segment Addition Postulate, $\underline{SR} + \underline{RV} = SV$.

$\underline{16} + RV = \underline{24}$ **Substitute for *SR* and *SV*.**

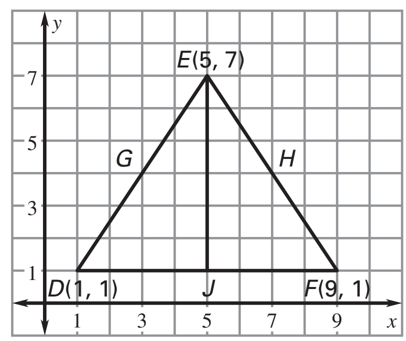
$RV = \underline{8}$ **Simplify.**

Answer So, $SV = \underline{24}$ and $RV = \underline{8}$.

Example 2 *Finding the Centroid of a Triangle*

Find the coordinates of centroid *C* of $\triangle DEF$.

Solution



You know that the centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

Choose the median $\overline{EJ}$. Find the coordinates of *J*, the midpoint of $\overline{DF}$. The coordinates of *J* are:

Use the Midpoint Formula to find the coordinates of *J*.

$$\left(\frac{\boxed{1} + \boxed{9}}{\boxed{2}}, \frac{\boxed{1} + \boxed{1}}{\boxed{2}}\right) = (\underline{5}, \underline{1})$$

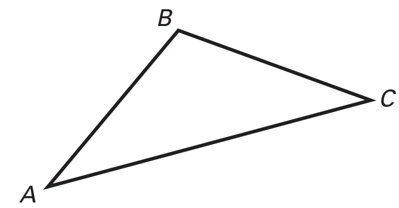
Find the distance from vertex *E* to midpoint *J*. The distance from *E* to *J* is $\underline{7} - \underline{1}$, or $\underline{6}$ units.

Determine the coordinates of centroid *C*, which is $\underline{\frac{1}{3}} \cdot \underline{6}$, or $\underline{2}$ units up from point *J* along the median $\overline{EJ}$.

Answer The coordinates of centroid *C* are $(\underline{5}, \underline{1} + \underline{2})$, or $(\underline{5}, \underline{3})$.

Example 3 ***Drawing Altitudes and Orthocenters***

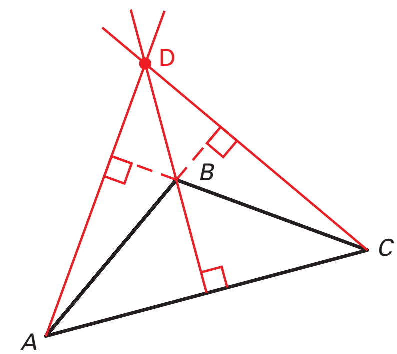
Is the orthocenter for $\triangle ABC$ located *inside*, *outside*, or *on* the triangle?



Solution

The orthocenter of $\triangle ABC$ is the intersection of the lines containing the altitudes of the triangle.

Use the diagram at the right to locate the orthocenter *D*.



Answer So, the orthocenter is located <u>outside</u> $\triangle ABC$.

Checkpoint Complete the following exercises.

1. *T* is the centroid of $\triangle QRS$ and $TU = 7$. Find *QU* and *QT*.

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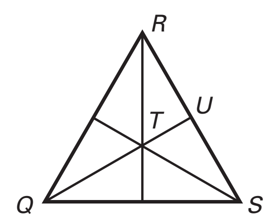

2. Is the orthocenter for an isosceles triangle *always* located inside the triangle? Explain.

No; The orthocenter for a right isosceles triangle is located on the triangle. The orthocenter for an obtuse isosceles triangle is located outside the triangle.