# 5.4 Midsegment Theorem

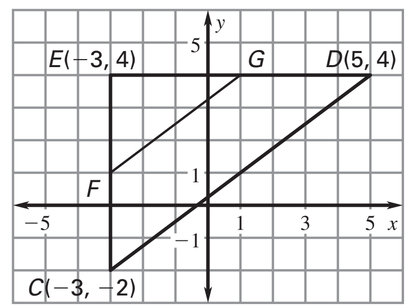
**Goals**
- Identify the midsegments of a triangle.
- Use properties of midsegments of a triangle.

**VOCABULARY**

Midsegment of a triangle A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

**Example 1** ***Using Midsegments***

Show that the midsegment $\overline{FG}$ is parallel to side $\overline{CD}$ and is half as long as $\overline{CD}$.




**Solution**

Use the Midpoint Formula to find the coordinates of $F$ and $G$.

$$F = \left(\frac{-3 + (-3)}{2}, \frac{4 + (-2)}{2}\right) = (-3, 1)$$

$$G = \left(\frac{-3 + 5}{2}, \frac{4 + 4}{2}\right) = (1, 4)$$

Next, find the slopes of $\overline{CD}$ and $\overline{FG}$.

$$\text{Slope of } \overline{CD} = \frac{4 - (-2)}{5 - (-3)} = \frac{6}{8} = \frac{3}{4}$$

$$\text{Slope of } \overline{FG} = \frac{4 - 1}{1 - (-3)} = \frac{3}{4}$$

Remember: The slope $m$ of the line passing through $(x_1, y_1)$ and $(x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

▶ Because the slopes are equal, $\overline{FG}$ is parallel to $\overline{CD}$.

Next, find the lengths of $\overline{CD}$ and $\overline{FG}$.

Use the Distance Formula to find the lengths.

$$CD = \sqrt{[5 - (-3)]^2 + [4 - (-2)]^2} = \sqrt{100} = 10$$

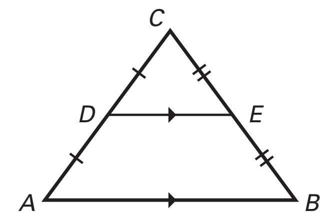
$$FG = \sqrt{[1 - (-3)]^2 + (4 - 1)^2} = \sqrt{25} = 5$$

▶ Because $\frac{FG}{CD} = \frac{5}{10} = \frac{1}{2}$, $\overline{FG}$ is half as long as $\overline{CD}$.

**THEOREM 5.9: MIDSEGMENT THEOREM**

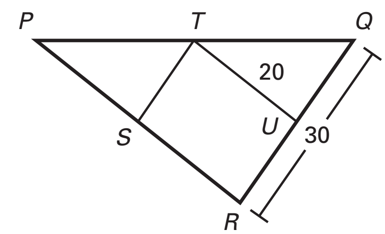
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

$\overline{DE} \parallel$ __$\overline{AB}$__ and $DE = \frac{1}{2}$ __$AB$__




**Example 2** ***Using the Midsegment Theorem***

$\overline{ST}$ and $\overline{TU}$ are midsegments of $\triangle PQR$. Find $PR$ and $ST$.



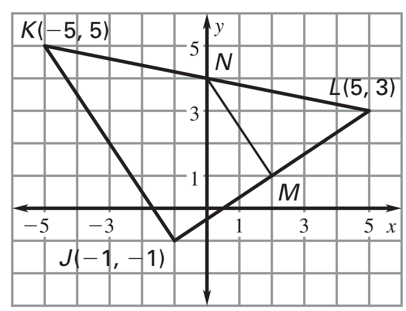

**Solution**

$PR = 2($ __$TU$__ $) = 2($ __20__ $) =$ __40__

$ST = \frac{1}{2}($ __$QR$__ $) = \frac{1}{2}($ __30__ $) =$ __15__

✔ ***Checkpoint*** **Complete the following exercises.**

**1.** Show that the midsegment $\overline{MN}$ is parallel to side $\overline{JK}$ and is half as long as $\overline{JK}$.

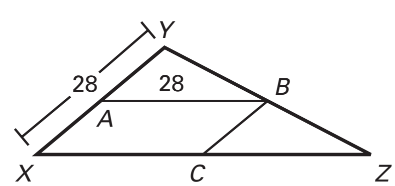



$\overline{MN}$ and $\overline{JK}$ each have a slope of $-\frac{3}{2}$. So, they are parallel.

Because $\frac{MN}{JK} = \frac{\sqrt{13}}{2\sqrt{13}} = \frac{1}{2}$, $\overline{MN}$ is half as long as $\overline{JK}$.

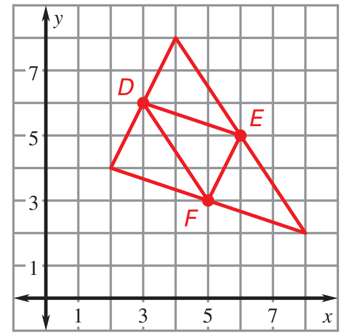
**2.** $\overline{AB}$ and $\overline{BC}$ are midsegments of $\triangle XYZ$. Find $XZ$ and $BC$.

56; 14



## Example 3 *Using Midpoints to Draw a Triangle*

The midpoints of the sides of a triangle are *D*(3, 6), *E*(6, 5), and *F*(5, 3). What are the coordinates of the vertices of the triangle?




**Solution**

1. **Plot** the midpoints.
2. **Connect** the midpoints to form $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$.
3. **Find** the slopes of the midsegments.

Find the slope using the graph and $m = \frac{\text{rise}}{\text{run}}$.

Slope of $\overline{DE}$ = $-\frac{1}{3}$ Slope of $\overline{EF}$ = 2

Slope of $\overline{DF}$ = $-\frac{3}{2}$

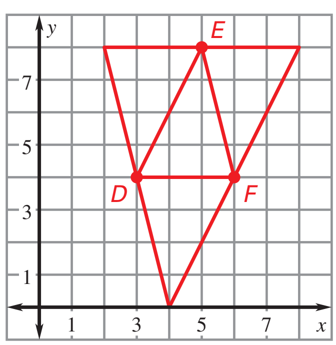
Each midsegment contains two of the unknown triangle's midpoints and is parallel to the side that contains the third midpoint. So, you know a point on each side of the triangle and the slope of each side.

4. **Draw** the lines that contain the three sides of the triangle.
5. **Identify** the points at which the lines intersect.

**Answer** The vertices of the triangle are (2, 4), (4, 8), and (8, 2).

**Checkpoint** Complete the following exercise.

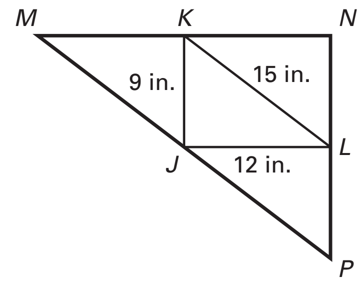
3. The midpoints of the sides of a triangle are *D*(3, 4), *E*(5, 8), and *F*(6, 4). What are the coordinates of the vertices of the triangle?




(2, 8), (8, 8), (4, 0)

## Example 4 *Perimeter of a Triangle*

$\overline{JK}$, $\overline{KL}$, and $\overline{JL}$ are midsegments of $\triangle MNP$. How does the perimeter of $\triangle MNP$ compare to the perimeter of $\triangle JKL$?




**Solution**

The lengths of the sides of $\triangle MNP$ are twice the lengths of the midsegments.

$MP = 2(\underline{KL}) = 2(\underline{15}) = \underline{30}$ in.

$MN = 2(\underline{JL}) = 2(\underline{12}) = \underline{24}$ in.

$NP = 2(\underline{KJ}) = 2(\underline{9}) = \underline{18}$ in.

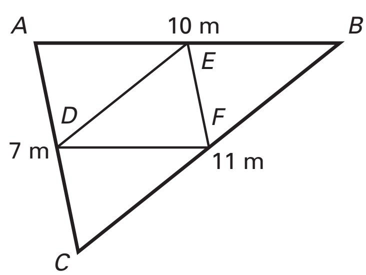
The perimeter of $\triangle MNP$ is $\underline{30} + \underline{24} + \underline{18} = \underline{72}$ inches.

The perimeter of $\triangle JKL$ is $\underline{15} + \underline{12} + \underline{9} = \underline{36}$ inches.

**Answer** The perimeter of $\triangle MNP$ is <u>twice</u> the perimeter of $\triangle JKL$.

**✔ *Checkpoint*** **Complete the following exercise.**

**4.** $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$ are midsegments of $\triangle ABC$. Find the perimeters of $\triangle ABC$ and $\triangle DEF$. How does the perimeter of $\triangle DEF$ compare to the perimeter of $\triangle ABC$?




28 m; 14 m; The perimeter of $\triangle DEF$ is half the perimeter of $\triangle ABC$.