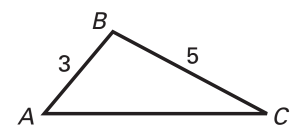
5.5 Inequalities in One Triangle

Goals
- Use triangle measurements to decide which side is longest or which angle is largest.
- Use the Triangle Inequality.

THEOREM 5.10

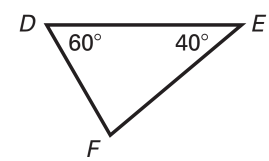
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$m\angle A > m\angle C$

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

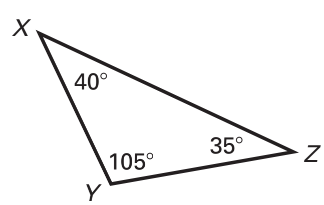


$EF > DF$

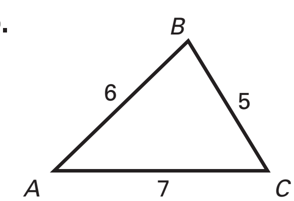
Example 1 ***Writing Measurements in Order from Least to Greatest***

Write the measures of the triangles in order from least to greatest.

a.



b.



Solution

a. $m\angle Z < m\angle X < m\angle Y$

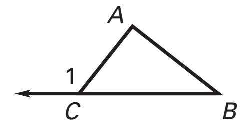
$XY < YZ < XZ$

b. $BC < AB < AC$

$m\angle A < m\angle C < m\angle B$

THEOREM 5.12: EXTERIOR ANGLE INEQUALITY

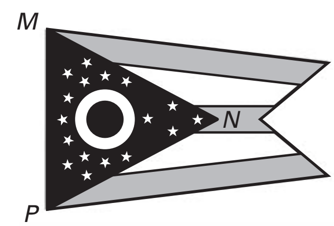
The measure of an exterior angle of a triangle is greater than the measure of either of the two nonadjacent interior angles.



$m\angle 1 > m\angle$ A and $m\angle 1 > m\angle$ B

Example 2 *Using Theorem 5.10*

State Flags The state flag of Ohio is shown at the right. In the flag, $\overline{MN} \cong \overline{PN}$ and $MP < MN$. What can you conclude about the angle measures in $\triangle MNP$?



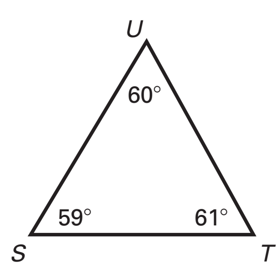
Solution

The sum of the angle measures in a triangle is 180°. In $\triangle MNP$, use logical reasoning to decide whether an angle measure is less than 60° or greater than 60°.

- Because $\overline{MN} \cong \overline{PN}$, $\triangle MNP$ is isosceles. So, $\angle$ M $\cong \angle$ P. Therefore, $m\angle$ M $= m\angle$ P.
- By Theorem 5.10, because $MP < MN$, $m\angle$ N $< m\angle$ P.
- Because $\overline{MN} \cong \overline{PN}$, $MN = PN$. So, by substitution, MP $< PN$. By Theorem 5.10, $m\angle$ N $< m\angle$ M.
- In addition, you can conclude that $m\angle M$ > 60°, $m\angle N$ < 60°, and $m\angle P$ > 60°.

Checkpoint **Write the measures of the triangle in order from least to greatest.**

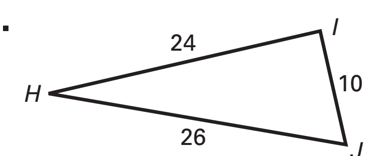
1.



$m\angle S < m\angle U < m\angle T$

$UT < ST < SU$

2.

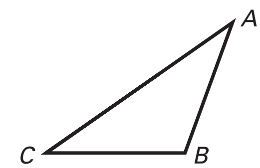


$IJ < HI < HJ$

$m\angle H < m\angle J < m\angle I$

THEOREM 5.13: TRIANGLE INEQUALITY

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$\underline{AB} + \underline{BC} > AC$

$AC + \underline{BC} > \underline{AB}$

$\underline{AB} + AC > \underline{BC}$

Example 3 ***Finding Possible Side Lengths***

A triangle has one side of 12 inches and another side of 20 inches. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Using the Triangle Inequality, you can write and solve inequalities.

$x + \underline{12} > \underline{20}$ $\underline{12} + \underline{20} > x$

$x > \underline{8}$ $\underline{32} > x$

Answer The length of the third side must be greater than $\underline{8}$ inches and less than $\underline{32}$ inches.

✔ ***Checkpoint*** **Decide if it is possible to construct a triangle having the given side lengths. If it is not possible, explain.**

3. 13 mm, 25 mm, 14 mm yes	**4.** 9 in., 17 in., 8 in. No; 9 + 8 is not greater than 17.

5. A triangle has one side of 8 millimeters and another side of 11 millimeters. Describe the possible lengths of the third side.

The length of the third side must be greater than 3 millimeters and less than 19 millimeters.