5.6 Indirect Proof and Inequalities in Two Triangles

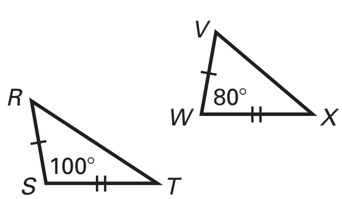
Goals
- Read and write an indirect proof.
- Use the Hinge Theorem and its converse to compare side lengths and angle measures.

VOCABULARY

Indirect proof An indirect proof is a proof in which you prove that a statement is true by first assuming that its opposite is true. If this assumption leads to an impossibility, then you have proved that the original statement is true.

THEOREM 5.14: HINGE THEOREM

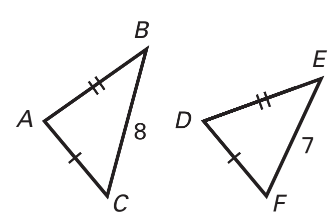
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.



$RT >$ VX

THEOREM 5.15: CONVERSE OF THE HINGE THEOREM

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



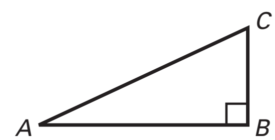
$m\angle A > m\angle$ D

GUIDELINES FOR WRITING AN INDIRECT PROOF

1. Identify the statement that you want to prove is true.
2. Begin by assuming the statement is false; assume the opposite is true.
3. Obtain statements that logically follow from the assumption.
4. If you obtain a contradiction, then the original statement must be true.

Example 1 *Using Indirect Proof*

Use an indirect proof to prove that a right triangle cannot have an obtuse angle.



Solution

Given: $\triangle ABC$ is a right triangle with $m\angle B = 90°$.

Prove: $\triangle ABC$ does not have an obtuse angle.

Begin by assuming that $\triangle ABC$ does have an obtuse angle.

$m\angle A > 90°$	**Assumption**
$m\angle A + m\angle B > 180°$	**Add angle measures.**

The sum of the measures of all *three* angles is 180°.

$m\angle A + m\angle B + m\angle C = 180°$	**Triangle Sum Theorem**
$m\angle A + m\angle B = 180° - m\angle C$	**Subtraction property of equality**

Substitute $180° - m\angle C$ for $m\angle A + m\angle B$.

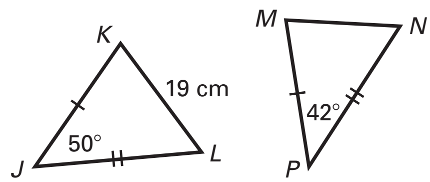
$180° - m\angle C > 180°$	**Substitution property of equality**
$0 > m\angle C$	**Simplify.**

The last statement is not possible because angle measures in triangles cannot be negative. So, you can conclude that the assumption must be false and that a right triangle cannot have an obtuse angle.

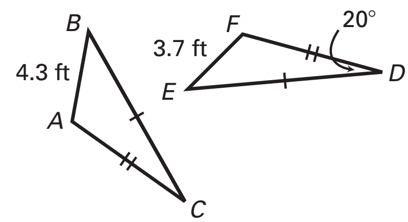
Example 2 *Finding Possible Side Lengths and Angle Measures*

You can use the Hinge Theorem and its converse to choose possible side lengths or angle measures from a given list.

a. Which of the following is a possible length for $\overline{MN}$: 16 cm, 19 cm, or 22 cm?



b. Which of the following is a possible measure for $\angle C$: 15°, 20°, or 25°?



Solution

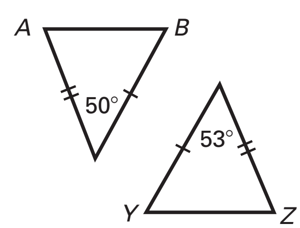
a. Because the included angle in $\triangle JKL$ is larger than the included angle in $\triangle MNP$, the third side $\overline{KL}$ must be longer than $\overline{MN}$.

So, of the three choices, the only possible length for $\overline{MN}$ is 16 centimeters.

b. Because the third side in $\triangle ABC$ is longer than the third side in $\triangle DEF$, the included $\angle C$ must be larger than $\angle D$. So, of the three choices, the only possible measure for $\angle C$ is 25°.

Checkpoint **Complete the statement with < or >.**

1. AB < YZ



2. $m\angle 1$ > $m\angle 2$