

# 6.1 Polygons

- Goals**
- Identify, name, and describe polygons.
  - Use the sum of the measures of the interior angles of a quadrilateral.

## VOCABULARY

**Polygon** A polygon is a plane figure that meets the following conditions: (1) It is formed by three or more segments called sides, such that no two sides with a common endpoint are collinear. (2) Each side intersects exactly two other sides, one at each endpoint.

**Sides** The sides of a polygon are the segments that form the polygon.

**Vertex** A vertex of a polygon is an endpoint of a side of the polygon.

**Convex** A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.

**Nonconvex** A nonconvex polygon is a polygon that is not convex.

**Concave** A concave polygon is a polygon that is not convex.

**Equilateral** A polygon is equilateral if all of its sides are congruent.

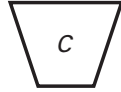
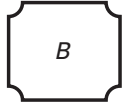
**Equiangular** A polygon is equiangular if all of its interior angles are congruent.

**Regular** A polygon is regular if it is equilateral and equiangular.

**Diagonal** A diagonal of a polygon is a segment that joins two nonconsecutive vertices.

**Example 1** Identifying Polygons

State whether the figure is a polygon. If it is not, explain why.

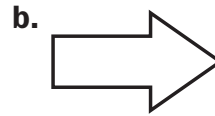
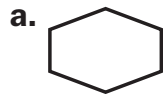
**Solution**

Figures C and D are polygons.

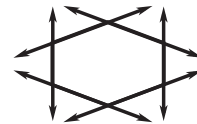
- Figure A is not a polygon because one of its sides intersects more than two other sides.
- Figure B is not a polygon because some of its sides are not segments.
- Figure E is not a polygon because two of the sides intersect only one other side.

**Example 2** Identifying Convex and Concave Polygons

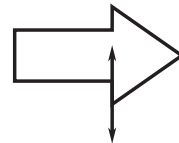
Identify the polygon and state whether it is convex or concave.

**Solution**

- a. The polygon has 6 sides, so it is a hexagon. When extended, none of the sides intersect the interior, so the polygon is convex.

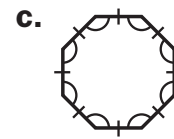
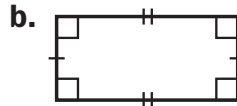
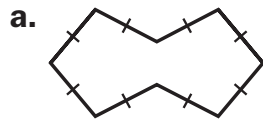


- b. The polygon has 7 sides, so it is a heptagon. When extended, some of the sides intersect the interior, so the polygon is concave.



**Example 3** Identifying Regular Polygons

Decide whether the polygon is regular.

**Solution**

- a. The polygon is equilateral, but it is not equiangular. So, it is not regular.
- b. The polygon is equiangular, but it is not equilateral. So, it is not regular.
- c. The polygon is equilateral and equiangular. So, it is regular.

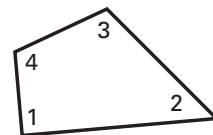
✓ **Checkpoint** Name the polygon. Is the polygon convex or concave? Is it regular?

<p>1. </p> <p>quadrilateral; convex; no</p>	<p>2. </p> <p>pentagon; concave; no</p>	<p>3. </p> <p>hexagon; convex; yes</p>
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**THEOREM 6.1: INTERIOR ANGLES OF A QUADRILATERAL**

The sum of the measures of the interior angles of a quadrilateral is  $360^\circ$ .

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = \underline{360^\circ}$$



**Example 4** Interior Angles of a QuadrilateralFind  $m\angle U$  and  $m\angle V$ .**Solution**

Find the value of  $x$ . Use the sum of the measures of the interior angles to write an equation involving  $x$ . Then, solve the equation.

$$5x^\circ + (3x + 10)^\circ + 72^\circ + 118^\circ = \underline{360}^\circ$$

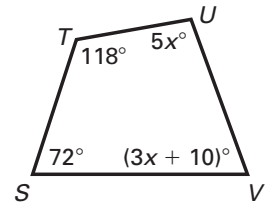
$$\underline{8x} + \underline{200} = \underline{360}$$

$$\underline{8x} = \underline{160}$$

$$x = \underline{20}$$

Use Theorem 6.1.

Combine like terms.

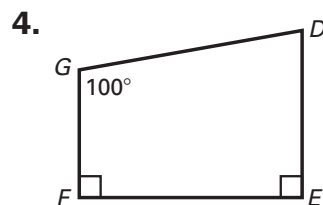
Subtract 200 from each side.Divide each side by 8.Find  $m\angle U$  and  $m\angle V$ .

$$m\angle U = 5x^\circ = (5 \cdot \underline{20})^\circ = \underline{100}^\circ$$

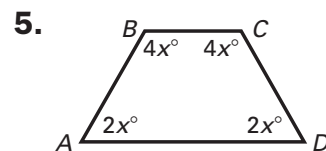
$$m\angle V = (3x + 10)^\circ = (3 \cdot \underline{20} + 10)^\circ = \underline{70}^\circ$$

Answer So,  $m\angle U = \underline{100}^\circ$  and  $m\angle V = \underline{70}^\circ$ .

✓ **Checkpoint** Find  $m\angle D$ .



$80^\circ$



$60^\circ$