6.2 Properties of Parallelograms

Goals
- Use some properties of parallelograms.
- Use properties of parallelograms in real-life situations.

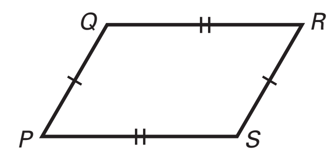
VOCABULARY

Parallelogram A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

THEOREM 6.2

If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

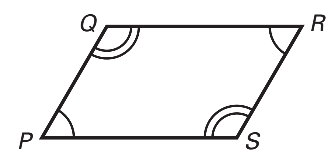
$\overline{PQ} \cong \overline{RS}$ and $\overline{SP} \cong \overline{QR}$



THEOREM 6.3

If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

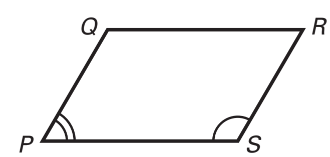
$\angle P \cong \angle R$ and $\angle Q \cong \angle S$



THEOREM 6.4

If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.

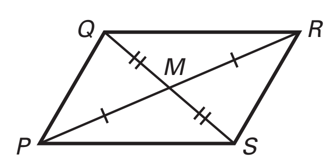
$m\angle P + m\angle Q = 180°$,
$m\angle Q + m\angle R = 180°$,
$m\angle R + m\angle S = 180°$,
$m\angle S + m\angle P = 180°$



THEOREM 6.5

If a quadrilateral is a parallelogram, then its diagonals **bisect** each other.

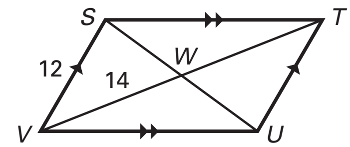
$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$



Example 1 *Using Properties of Parallelograms*

STUV is a parallelogram. Find the unknown length.

a. *TU* **b.** *WT*



Solution

a. $TU = \underline{SV}$ — Opposite sides of a ▱ are ≅ .

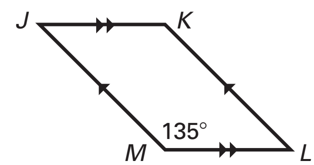
$TU = \underline{12}$ — Substitute $\underline{12}$ for $\underline{SV}$.

b. $WT = \underline{WV}$ — Diagonals of a ▱ bisect each other.

$WT = \underline{14}$ — Substitute $\underline{14}$ for $\underline{WV}$.

Example 2 *Using Properties of Parallelograms*

JKLM is a parallelogram. Find $m\angle L$.



Solution

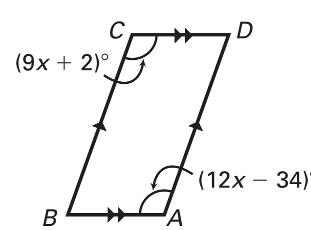
$m\angle L + m\angle \underline{M} = \underline{180}°$ — Consecutive angles of a ▱ are supplementary.

$m\angle L + \underline{135}° = \underline{180}°$ — Substitute $\underline{135}°$ for $m\angle \underline{M}$.

$m\angle L = \underline{45}°$ — Subtract $\underline{135}°$ from each side.

Example 3 *Using Algebra with Parallelograms*

ABCD is a parallelogram. Find the value of *x*.



Solution

$m\angle \underline{C} = m\angle \underline{A}$ — Opposite angles of a ▱ are ≅.

$\underline{9x} + \underline{2} = \underline{12x} - \underline{34}$ — Substitute.

$\underline{9x} + \underline{36} = \underline{12x}$ — Add $\underline{34}$ to each side.

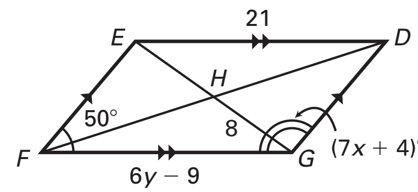
$\underline{36} = \underline{3}\,x$ — Subtract $\underline{9x}$ from each side.

$\underline{12} = x$ — Divide each side by $\underline{3}$.

✔ ***Checkpoint*** **Find the measure or value in parallelogram *DEFG*. Explain your reasoning.**

1. Find $m\angle D$.

50° ; Opposite ∠s of a ▱ are ≅.



2. Find *EH*.

8; Diagonals of a ▱ bisect each other.

3. Find the value of *y* in the parallelogram above.

5; opposite sides of a ▱ are ≅.

4. Find the value of *x* in the parallelogram above.

18; Consecutive ∠s of a ▱ are supplementary.

Example 4 ***Using Parallelograms in Real Life***

Gemstones A gemstone is cut so that one of its facets has four sides. The measures of the consecutive angles in the facet are 45°, 135°, 135°, and 45°. Is the facet a parallelogram? Explain.

Solution

The facet is not a parallelogram. Here are two reasons why.

- The opposite angles __are not congruent__.
- The sums of the measures of the consecutive angles are __180__°, __270__°, __180__°, and __90__°. If the facet were a parallelogram, then all pairs of consecutive angles would be __supplementary__.