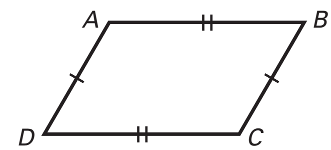
6.3 Proving Quadrilaterals are Parallelograms

Goals
- Prove that a quadrilateral is a parallelogram.
- Use coordinate geometry with parallelograms.

THEOREM 6.6

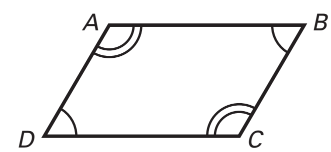
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

THEOREM 6.7

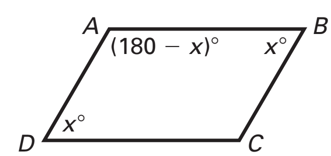
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

THEOREM 6.8

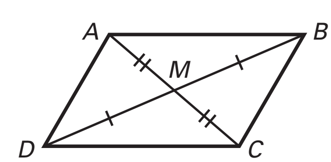
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

THEOREM 6.9

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

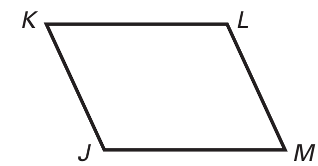


ABCD is a parallelogram.

Example 1 ***Proof of Theorem 6.8***

Prove Theorem 6.8.

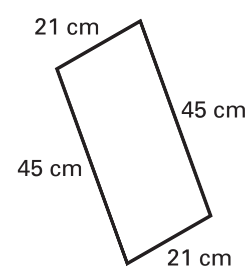
Given: $\angle J$ is supplementary to $\angle K$ and $\angle M$.
Prove: *JKLM* is a parallelogram.



Statements	Reasons
1. $\angle J$ is supplementary to $\angle K$.	1. Given
2. $\overline{JM} \parallel \overline{KL}$	2. Consecutive Interior ∠s Converse
3. $\angle J$ is supplementary to $\angle M$.	3. Given
4. $\overline{JK} \parallel \overline{ML}$	4. Consecutive Interior ∠s Converse
5. *JKLM* is a parallelogram.	5. Definition of a parallelogram

Checkpoint **Complete the following exercise.**

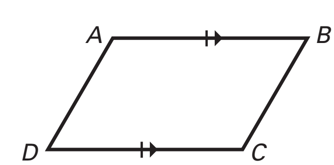
1. **Stained Glass** A pane in a stained glass window has the shape shown at the right. How do you know that the pane is a parallelogram?



The pane is a quadrilateral and both pairs of opposite sides are congruent. So, by Theorem 6.6, the quadrilateral is a parallelogram.

THEOREM 6.10

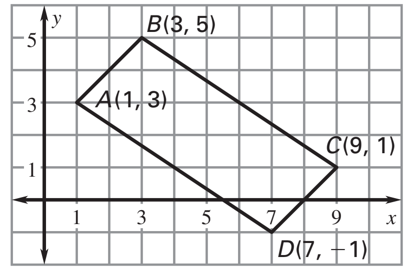
If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



ABCD is a parallelogram.

Example 2 *Using Properties of Parallelograms*

Show that $A(1, 3)$, $B(3, 5)$, $C(9, 1)$, and $D(7, -1)$ are the vertices of a parallelogram.



Solution

Method 1 Show that opposite sides have the same slope.

To find the slopes of the opposite sides, use

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Slope of $\overline{AB}$:

$$\frac{5 - 3}{3 - 1} = 1$$

Slope of $\overline{BC}$:

$$\frac{5 - 1}{3 - 9} = -\frac{2}{3}$$

Slope of $\overline{CD}$:

$$\frac{1 - (-1)}{9 - 7} = 1$$

Slope of $\overline{AD}$:

$$\frac{3 - (-1)}{1 - 7} = -\frac{2}{3}$$

▶ $\overline{AB}$ and $\overline{CD}$ have the same slope, so they are parallel. Similarly, $\overline{BC} \parallel \overline{AD}$. Because opposite sides are parallel, *ABCD* is a parallelogram.

Method 2 Show that opposite sides have the same length.

To find the lengths of the sides, use the Distance Formula.

$$AB = \sqrt{(3 - 1)^2 + (5 - 3)^2} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(9 - 3)^2 + (1 - 5)^2} = \sqrt{52} = 2\sqrt{13}$$

$$CD = \sqrt{(9 - 7)^2 + [1 - (-1)]^2} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(7 - 1)^2 + (-1 - 3)^2} = \sqrt{52} = 2\sqrt{13}$$

▶ $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Because both pairs of opposite sides are congruent, *ABCD* is a parallelogram.

Method 3 Show that one pair of opposite sides is congruent and parallel.

Slope of $\overline{AB}$ = Slope of $\overline{CD}$ = 1

$AB = CD = 2\sqrt{2}$

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel. So, *ABCD* is a parallelogram.