

6.4

Rhombuses, Rectangles, and Squares

- Goals**
- Use properties of sides and angles of rhombuses, rectangles, and squares.
 - Use properties of diagonals of rhombuses, rectangles, and squares.

VOCABULARY

Rhombus A rhombus is a parallelogram with four congruent sides.

Rectangle A rectangle is a parallelogram with four right angles.

Square A square is a parallelogram with four congruent sides and four right angles.

Example 1 Describing a Special Parallelogram

Decide whether the statement is *always*, *sometimes*, or *never* true.

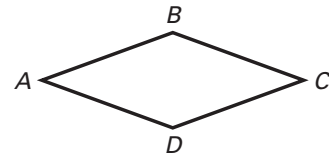
- A square is a rectangle.
- A rectangle is a square.

Solution

- The statement is always true. Because all squares have four right angles, squares are always rectangles.
- The statement is sometimes true. If a rectangle has four congruent sides, then it is also a square.

Example 2 Using Properties of Special Parallelograms

$ABCD$ is a rhombus. What else do you know about $ABCD$?



- $ABCD$ has four congruent sides.
- Its opposite sides are parallel.
- Its opposite angles are congruent.
- Its diagonals bisect each other.
- Its consecutive angles are supplementary.

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

RECTANGLE COROLLARY

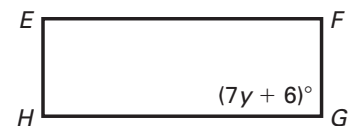
A quadrilateral is a rectangle if and only if it has four right angles.

SQUARE COROLLARY

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

Example 3 Using Properties of a Rectangle

In the diagram, $EFGH$ is a rectangle.
What is the value of y ?

**Solution**

All four angles of a rectangle are right angles. So, $m\angle G = 90^\circ$.

$$(7y + 6)^\circ = \underline{90}^\circ \quad \text{Write equation.}$$

$$7y = \underline{84} \quad \text{Subtract } \underline{6} \text{ from each side.}$$

$$y = \underline{12} \quad \text{Divide each side by } \underline{7}.$$

- ✓ **Checkpoint** Decide whether the statement is *always*, *sometimes*, or *never* true.

1. A rhombus is a square.

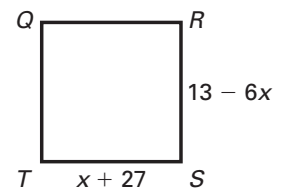
Sometimes

2. A parallelogram is a rectangle.

Sometimes

3. $QRST$ is a square. What is the value of x ?

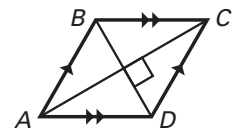
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THEOREM 6.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

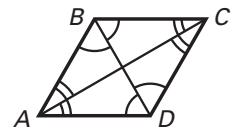
$ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



THEOREM 6.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

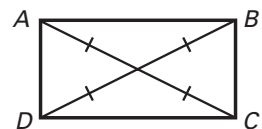
$ABCD$ is a rhombus if and only if \overline{AC} bisects $\angle DAB$ and $\angle BCD$ and \overline{BD} bisects $\angle ADC$ and $\angle ABC$.



THEOREM 6.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

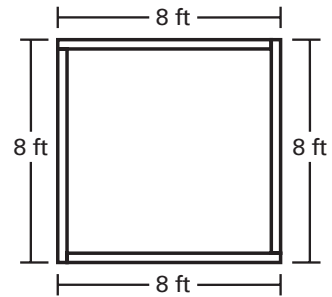
$ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



Example 4 *Checking a Square*

Tree House You are building a tree house.

- a. To make the base of the floor, you nail four pieces of wood together as shown at the right. What is the shape of the floor base? Explain.
- b. To make sure the base is a square, you measure the length of the diagonals. Both diagonals measure 11 feet 4 inches. Is the base a square? Explain.

**Solution**

- a. All four sides are congruent. So, the floor base is a rhombus.
- b. The diagonals are congruent. So, the floor base is a rectangle. Because the base is a rhombus and a rectangle, it is also a square.

✓ **Checkpoint** Decide whether the statement is *true* or *false*. Explain your reasoning.

4. The diagonals of a rhombus are always congruent.

False

5. The diagonals of a square are never perpendicular.

False

6. Each diagonal of a rectangle sometimes bisects a pair of opposite angles.

True