6.5 Trapezoids and Kites

Goals
- Use properties of trapezoids.
- Use properties of kites.

VOCABULARY

Trapezoid A trapezoid is a quadrilateral with exactly one pair of parallel sides.

Bases of a trapezoid The parallel sides of a trapezoid are called bases.

Base angles of a trapezoid A trapezoid has two pairs of base angles. Each pair shares a base as a side.

Legs of a trapezoid The nonparallel sides of a trapezoid are called legs.

Isosceles trapezoid An isosceles trapezoid is a trapezoid with congruent legs.

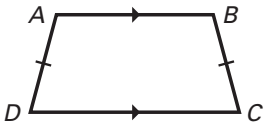
Midsegment of a trapezoid The midsegment of a trapezoid is the segment that connects the midpoints of its legs.

Kite A kite is a quadrilateral that has two pairs of consecutive congruent sides, but its opposite sides are not congruent.

THEOREM 6.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

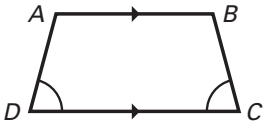
$\angle A \cong \angle B$, $\angle C \cong \angle D$



THEOREM 6.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

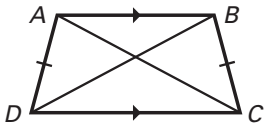
ABCD is an isosceles trapezoid.



THEOREM 6.16

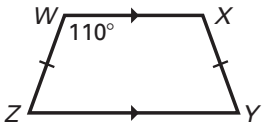
A trapezoid is isosceles if and only if its diagonals are congruent.

ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



Example 1 ***Using Properties of Isosceles Trapezoids***

WXYZ is an isosceles trapezoid. Find $m\angle X$, $m\angle Y$, and $m\angle Z$.



Solution

- *WXYZ* is an isosceles trapezoid, so $m\angle X = m\angle W = 110°$.
- $\angle W$ and $\angle Z$ are consecutive interior angles formed by parallel lines, so they are supplementary.

$m\angle W + m\angle Z = 180°$ **Consecutive Interior Angles Theorem**

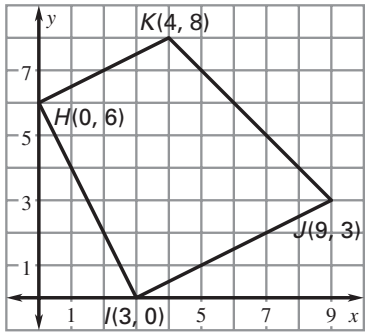
$110° + m\angle Z = 180°$ **Substitute for $m\angle W$.**

$m\angle Z = 70°$ **Subtract 110° from each side.**

- *WXYZ* is an isosceles trapezoid, so $m\angle Y = m\angle Z = 70°$.

Example 2 *Using Properties of Trapezoids*

Show that *HIJK* is a trapezoid.



Compare the slopes of opposite sides.

Slope of $\overline{HK} = \frac{8-6}{4-0} = \frac{2}{4} = \frac{1}{2}$

Slope of $\overline{IJ} = \frac{3-0}{9-3} = \frac{3}{6} = \frac{1}{2}$

The slopes of $\overline{HK}$ and $\overline{IJ}$ are equal, so $\overline{HK} \parallel \overline{IJ}$.

Slope of $\overline{HI} = \frac{6-0}{0-3} = \frac{6}{-3} = -2$

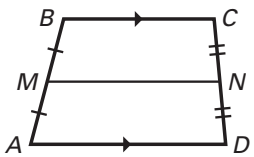
Slope of $\overline{JK} = \frac{8-3}{4-9} = \frac{5}{-5} = -1$

The slopes of $\overline{HI}$ and $\overline{JK}$ are not equal, so $\overline{HI}$ is not parallel to $\overline{JK}$.

Answer Because $\overline{HK} \parallel \overline{IJ}$ and $\overline{HI}$ is not parallel to $\overline{JK}$, *HIJK* is a trapezoid .

THEOREM 6.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

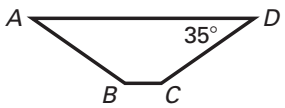
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.



$\overline{MN} \parallel \overline{AD}$, $\overline{MN} \parallel \overline{BC}$, $MN = \frac{1}{2}(AD + BC)$

✔ ***Checkpoint*** **Complete the following exercise.**

1. *ABCD* is an isosceles trapezoid. Find $m\angle A$, $m\angle B$, and $m\angle C$.

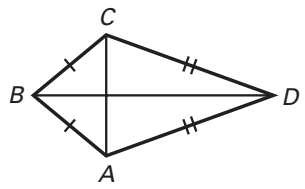


$m\angle A = 35°$
$m\angle B = m\angle C = 145°$

THEOREM 6.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

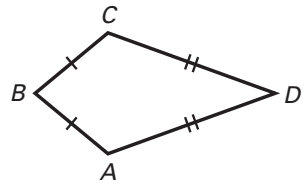
$\overline{AC} \perp \overline{BD}$



THEOREM 6.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

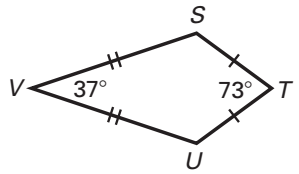
$\angle A \cong \angle C$, $\angle B \ncong \angle D$



Example 3 ***Angles of a Kite***

Find $m\angle S$ and $m\angle U$.

STUV is a kite, so $\angle S \cong \angle U$ and $m\angle S = m\angle U$.



$2(m\angle S) + m\angle T + m\angle V = 360°$ — **Sum of measures of int. $\angle$s of quad. is 360°.**

$2(m\angle S) + 73° + 37° = 360°$ — **Substitute.**

$2(m\angle S) = 250°$ — **Simplify.**

$m\angle S = 125°$ — **Divide each side by 2.**

Answer So, $m\angle S = m\angle U = 125°$.

Checkpoint **Complete the following exercise.**

2. Find $m\angle X$ and $m\angle Z$.

$m\angle X = m\angle Z = 62°$