6.6 Special Quadrilaterals

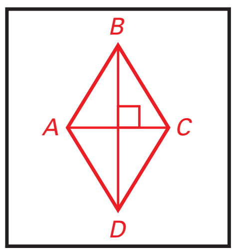
Goals
- Identify special quadrilaterals based on limited information.
- Prove that a quadrilateral is a special type of quadrilateral.

Example 1 ***Identifying Quadrilaterals***

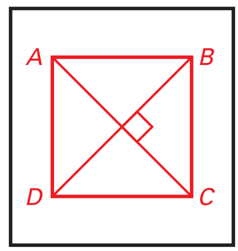
Quadrilateral *ABCD* has diagonals that are perpendicular. What types of quadrilaterals meet this condition?

Solution

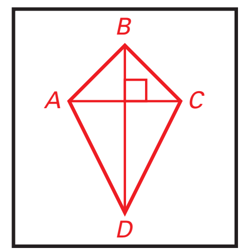
There are three types of quadrilaterals that meet this condition. Draw and label each type of quadrilateral.



rhombus

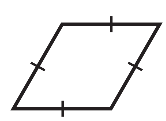


square

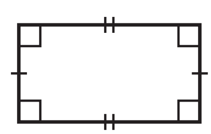


kite

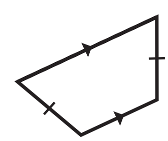
Checkpoint **Identify the special quadrilateral. Use the most specific name.**

1. 

rhombus

2. 

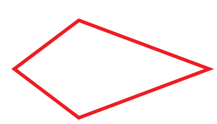
rectangle

3. 

isosceles trapeziod

4. A quadrilateral has diagonals that are not congruent. What types of quadrilaterals meet this condition? Draw and label each type of quadrilateral.

parallelogram



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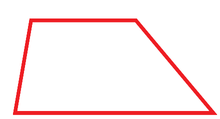
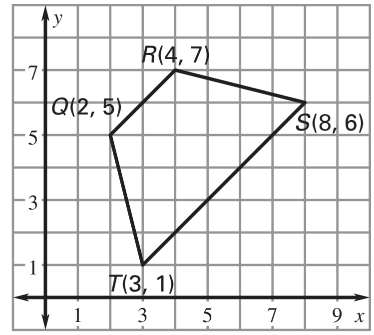


trapezoid

Example 2 *Proving a Quadrilateral is an Isosceles Trapezoid*

Show that *QRST* is an isosceles trapezoid.



Solution

Here is one way to show that *QRST* is an isosceles trapezoid.

1. Show that *QRST* is a trapezoid by proving $\overline{QR} \parallel$ $\overline{ST}$ and $\overline{QT}$ is not parallel to $\overline{RS}$.

$$\text{Slope of } \overline{QR} = \frac{7 - 5}{4 - 2} = 1$$

$$\text{Slope of } \overline{ST} = \frac{6 - 1}{8 - 3} = 1$$

$$\text{Slope of } \overline{QT} = \frac{5 - 1}{2 - 3} = -4$$

$$\text{Slope of } \overline{RS} = \frac{7 - 6}{4 - 8} = -\frac{1}{4}$$

Recall that the slopes of parallel lines are equal.

The slopes of $\overline{QR}$ and $\overline{ST}$ are equal, so $\overline{QR} \parallel \overline{ST}$. The slopes of $\overline{QT}$ and $\overline{RS}$ are not equal. So, these segments are not parallel.

2. Show that *QRST* is isosceles by proving $\overline{QT} \cong \overline{RS}$.

$$QT = \sqrt{(2 - 3)^2 + (5 - 1)^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$$

$$RS = \sqrt{(4 - 8)^2 + (7 - 6)^2} = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$$

Use the Distance Formula to find the lengths of the segments.

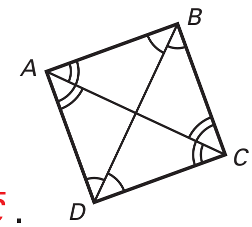
Because $QT = RS$, $\overline{QT} \cong \overline{RS}$.

Answer Because *QRST* is a quadrilateral with exactly one pair of parallel sides, it is a trapezoid. Because its legs are congruent, *QRST* is an isosceles trapezoid.

Example 3 *Identifying a Quadrilateral*

Use the Alternate Interior Angles Converse to make these conclusions.

What type of quadrilateral is *ABCD*? Explain your reasoning.



Solution

$\angle BAC \cong \angle ACD$, so you can conclude that $\overline{AB} \parallel \overline{CD}$. Similarly, $\angle ADB \cong \angle DBC$, so $\overline{AD} \parallel \overline{BC}$.

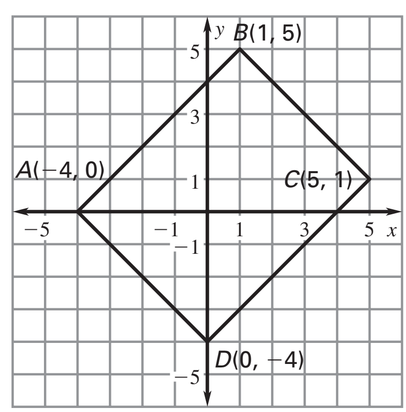
- Because *ABCD* is a quadrilateral with both pairs of opposite sides parallel, *ABCD* is a parallelogram.
- Because *ABCD* is a parallelogram and each diagonal bisects a pair of opposite angles, *ABCD* is a rhombus.

Checkpoint Complete the following exercises.

5. In Example 3, can you conclude that *ABCD* is a square? Explain.

No; *ABCD* is a square only if $m\angle A = m\angle B = m\angle C = m\angle D = 90°$. But these angle measures are not known.

6. What type of quadrilateral is *ABCD*? Explain how to prove it.



Rectangle; *Sample Answer:* Show that *ABCD* is a parallelogram by proving that $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. Then show that the diagonals $\overline{AC}$ and $\overline{BD}$ are congruent.