6.7 Areas of Triangles and Quadrilaterals

Goals • Find the areas of squares, rectangles, parallelograms, and triangles.
• Find the areas of trapezoids, kites, and rhombuses.

POSTULATE 22: AREA OF A SQUARE POSTULATE

The area of a square is the square of the length of its side, or $A =$ _s^2_ .

POSTULATE 23: AREA CONGRUENCE POSTULATE

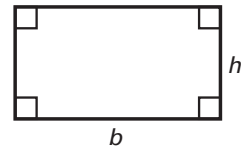If two polygons are _congruent_ , then they have the same area.

POSTULATE 24: AREA ADDITION POSTULATE

The area of a region is the _sum_ of the areas of its nonoverlapping parts.

THEOREM 6.20: AREA OF A RECTANGLE

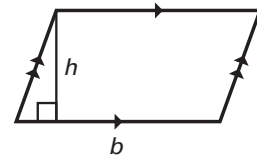The area of a rectangle is the product of its base and height.

$A =$ _bh_

THEOREM 6.21: AREA OF A PARALLELOGRAM

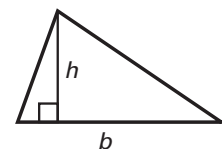The area of a parallelogram is the product of a base and its corresponding height.
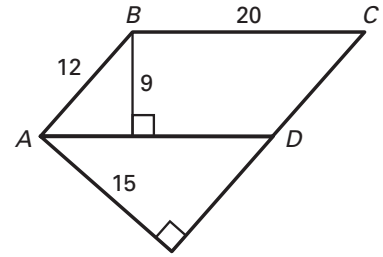
$A =$ _bh_

THEOREM 6.22: AREA OF A TRIANGLE

The area of a triangle is one half the product of a base and its corresponding height.

$A =$ _$\frac{1}{2}bh$_

Example 1   *Using the Area Theorems*

Find the area of ▱*ABCD*.

Solution

Use $\overline{AB}$ as the base. So, *b* = __12__ and *h* = __15__.

   Area = *bh*

        = __12__ (__15__)

        = __180__ square units

Notice that you get the same area using $\overline{BC}$ as the base.

Example 2   *Finding the Height of a Triangle*

Rewrite the formula for the area of a triangle in terms of *h*. Then use your formula to find the height of a triangle that has an area of 18 and a base length of 6.

Solution

Rewrite the area formula so *h* is alone on one side of the equation.

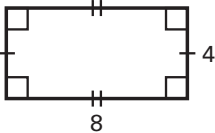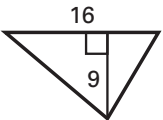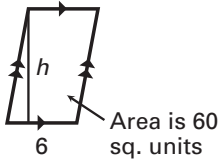 $A = \dfrac{1}{2}bh$ **Formula for the area of a triangle**

  __$2A$__ = __bh__   **Multiply each side by 2.**

  $\dfrac{2A}{b} = h$ **Divide each side by *b*.**

Substitute __18__ for *A* and __6__ for *b* to find the height of the triangle.

  $h = \dfrac{2A}{b} = \dfrac{2(18)}{6} = \underline{6}$

Answer The height of the triangle is __6__.

✔ *Checkpoint* **Find the area or height of the polygon.**

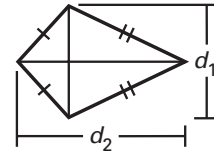| 1.	2.	3.
4, 8	16, 9	*h*, 6, Area is 60 sq. units
32 sq. units	**72 sq. units**	**10 units**

THEOREM 6.23: AREA OF A TRAPEZOID

The area of a trapezoid is one half the product of the height and the sum of the bases.

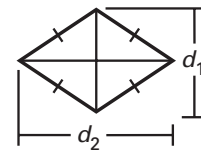$$A = \frac{1}{2}h(b_1 + b_2)$$
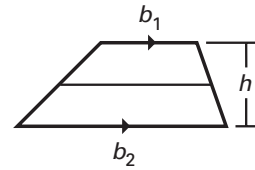
THEOREM 6.24: AREA OF A KITE

The area of a kite is one half the product of the lengths of its diagonals.

$$A = \frac{1}{2}d_1 d_2$$

THEOREM 6.25: AREA OF A RHOMBUS

The area of a rhombus is one half the product of the lengths of the diagonals.

$$A = \frac{1}{2}d_1 d_2$$

Example 3   *Finding the Area of a Trapezoid*

Find the area of trapezoid *JKLM*.

Solution

The height of *JKLM* is
$h =$ __8__ $-$ __2__ $=$ __6__.

Find the lengths of the bases.

$b_1 = JK =$ __7__ $-$ __0__ $=$ __7__

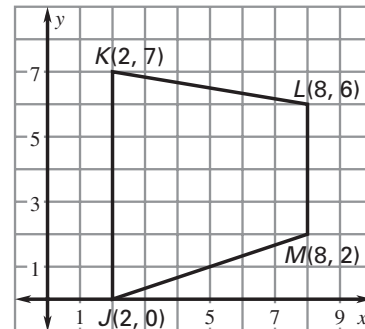$b_2 = LM =$ __6__ $-$ __2__ $=$ __4__

Substitute __6__ for h, __7__ for b_1, and __4__ for b_2 to find the area of the trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ **Formula for area of a trapezoid**

$= \frac{1}{2}(\,\underline{6}\,)(\,\underline{7}\, + \,\underline{4}\,)$ **Substitute.**

$= \underline{\,33\,}$ **Simplify.**

Answer The area of trapezoid *JKLM* is __33__ square units.

Example 4   *Finding the Area of a Rhombus*

Use the information in the diagram to find the area of rhombus *ABCD*.
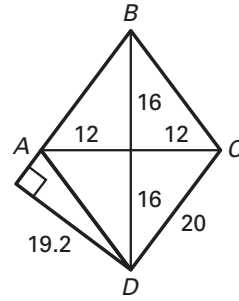
Solution

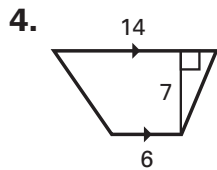**Method 1** Use the formula for the area of a rhombus. Let $d_1 = BD = $ _32_ and $d_2 = AC = $ _24_ .

$A = \dfrac{1}{2} d_1 d_2$

$ = \dfrac{1}{2} (\underline{32})(\underline{24})$

$ = \underline{384}$ square units

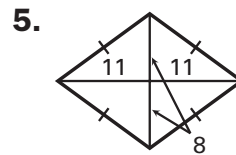**Method 2** Use the formula for the area of a parallelogram. Let $b = $ _20_ and $h = $ _19.2_ .

$A = bh$

$ = (\underline{20})(\underline{19.2})$

$ = \underline{384}$ square units

✔ *Checkpoint* Find the area of the polygon.

4.	5.	6.
70 sq. units	176 sq. units	360 sq. units