7.1 Rigid Motion in a Plane

Goals
- Identify the three basic rigid transformations.
- Use transformations in real-life situations.

VOCABULARY

Image An image is a new figure that results from the transformation of a figure in a plane.

Preimage A preimage is the original figure in the transformation of a figure in a plane.

Transformation A transformation is the operation that maps, or moves, a preimage onto an image.

Isometry An isometry is a transformation that preserves lengths. Isometries are also called rigid transformations.

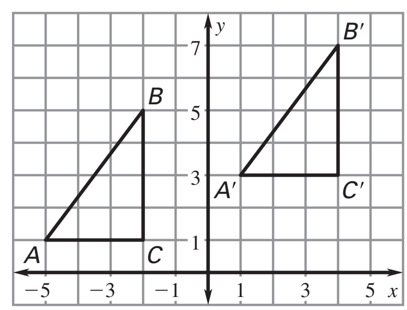
Example 1 ***Naming Transformations***

Use the graph of the transformation at the right.

a. Name and describe the transformation.

b. Name the coordinates of the vertices of the image.

c. Is $\triangle ABC$ congruent to its image?



When you name an image, take the corresponding point of the preimage and add a prime symbol. For instance, if the preimage is *A*, the image is *A*′, read as "*A* prime."

Solution

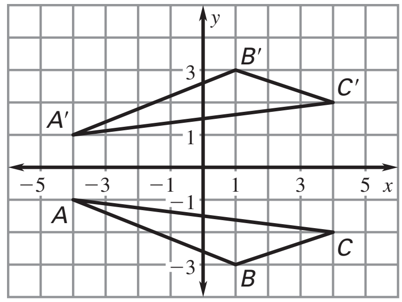
a. The transformation is a translation. You can imagine that the image was obtained by sliding $\triangle ABC$ up and to the right.

b. The coordinates of the vertices of the image, $\triangle A'B'C'$, are $A'(1, 3)$, $B'(4, 7)$, and $C'(4, 3)$.

c. Yes, $\triangle ABC$ is congruent to its image $\triangle A'B'C'$. One way to show this would be to use the Distance Formula to find the lengths of the sides of both triangles. Then use the SSS Congruence Postulate.

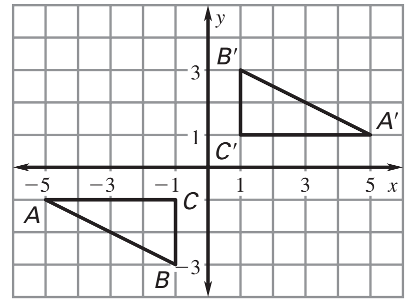
Checkpoint **Name and describe the transformation. Is $\triangle ABC$ congruent to its image?**

1.



Reflection in the *x*-axis; yes

2.

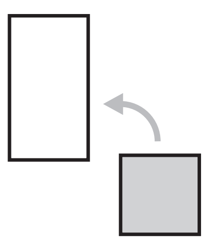


Rotation about the origin; yes

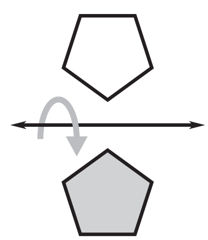
Example 2 *Identifying Isometries*

Does the transformation appear to be an isometry?

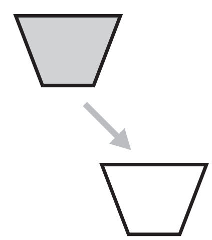
a.



b.



c.



Solution

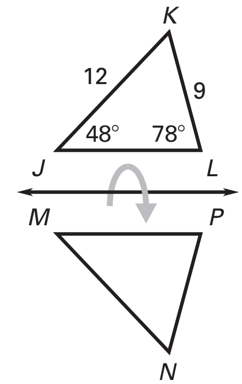
a. No. The image is not congruent to the preimage.

b. Yes. The shaded pentagon is reflected in a line to produce a congruent unshaded pentagon.

c. Yes. The shaded trapezoid is translated down and to the right to form a congruent unshaded trapezoid.

Example 3 *Preserving Length and Angle Measures*

In the diagram, $\triangle JKL$ is mapped onto $\triangle MNP$. The mapping is a reflection. Given that $\triangle JKL \rightarrow \triangle MNP$ is an isometry, find the length of $\overline{NP}$ and the measure of $\angle M$.



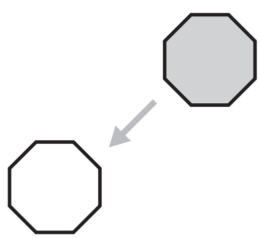
Solution

The statement "$\triangle JKL \rightarrow \triangle MNP$" implies that $J \rightarrow$ M, $K \rightarrow$ N, and $L \rightarrow$ P. Because the transformation is an isometry, the two triangles are congruent.

Answer So, $NP =$ KL $=$ 9 and $m\angle M = m\angle$ J $=$ 48°.

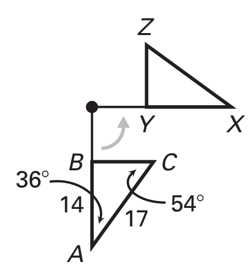
Checkpoint Complete the following exercises.

3. Does the transformation appear to be an isometry? Explain.



Yes; the shaded octagon is translated down and to the left to produce a congruent unshaded octagon.

4. $\triangle ABC$ is mapped onto $\triangle XYZ$. Given that $\triangle ABC \rightarrow \triangle XYZ$ is an isometry, find XZ and $m\angle Y$.



$XZ = 17$; $m\angle Y = 90°$