7.3 Rotations

Goals
- Identify rotations in a plane.
- Determine whether a figure has rotational symmetry.

VOCABULARY

Rotation A rotation is a type of transformation in which a figure is turned about a fixed point.

Center of rotation In a rotation, the fixed point is called the center of rotation.

Angle of rotation In a rotation, the angle of rotation is the angle formed when rays are drawn from the center of rotation to a point and its image.

Rotational symmetry A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation of 180° or less.

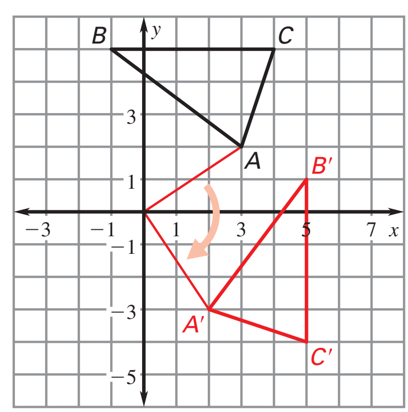
THEOREM 7.2: ROTATION THEOREM

A rotation is an isometry.

Example 1 ***Rotations in a Coordinate Plane***

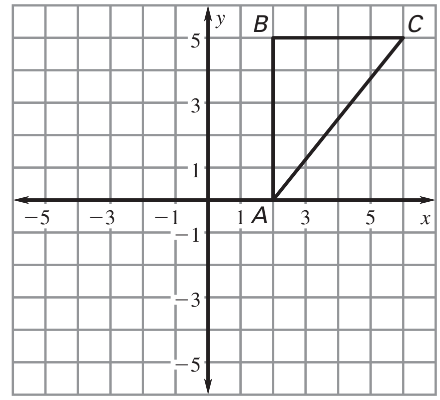
Rotate △*ABC* clockwise 90° about the origin and name the coordinates of the new vertices.

△*ABC* is shown in the graph. Use a protractor, a compass, and a straightedge to find the rotated vertices and draw △*A′B′C′*. The coordinates of △*ABC* are listed below. Write the coordinates of △*A′B′C′*.



△*ABC*	△*A′B′C′*
A(3, 2)	*A′*(2, −3)
B(−1, 5)	*B′*(5, 1)
C(4, 5)	*C′*(5, −4)

Checkpoint **Name the coordinates of the vertices of the image after the given rotation of $\triangle ABC$ about the origin.**



1. 90° clockwise

$A'(0, -2)$, $B'(5, -2)$, $C'(5, -6)$

2. 90° counterclockwise

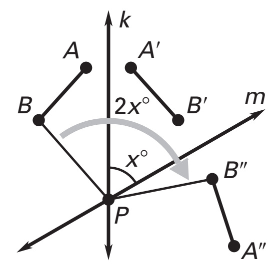
$A'(0, 2)$, $B'(-5, 2)$, $C'(-5, 6)$

THEOREM 7.3

If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is a rotation about point P.

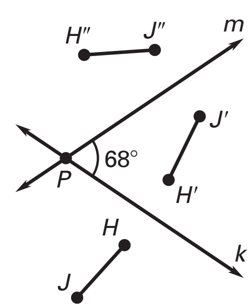
The angle of rotation is $\underline{2x}$°, where $x°$ is the measure of the acute or right angle formed by k and m.

$m\angle BPB'' = \underline{2x}°$



Example 2 ***Using Theorem 7.3***

In the diagram, $\overline{HJ}$ is reflected in line k to produce $\overline{H'J'}$. This segment is then reflected in line m to produce $\overline{H''J''}$. Describe the transformation that maps $\overline{HJ}$ to $\overline{H''J''}$.



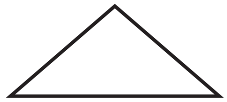
Solution

The acute angle between lines k and m has a measure of $\underline{68}$°. Applying Theorem 7.3, you can conclude that the transformation that maps $\overline{HJ}$ to $\overline{H''J''}$ is a counterclockwise rotation of $\underline{136}$° about point P.

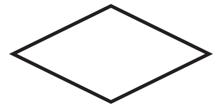
Example 3 *Identifying Rotational Symmetry*

Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself.

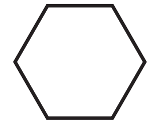
a. Isosceles triangle



b. Rhombus



c. Regular hexagon



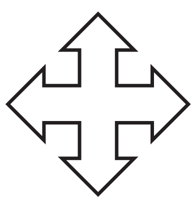
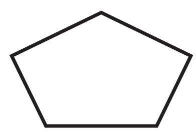
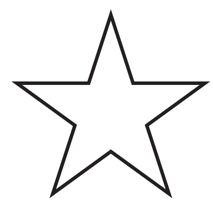
Solution

a. No. This isosceles triangle does not have rotational symmetry.

b. Yes. This rhombus has rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about its center.

c. Yes. This hexagon has rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 60°, 120°, or 180° about its center.

✔ ***Checkpoint*** **Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself.**

3.	4.	5.
 Yes; the figure can be mapped onto itself by a clockwise or counterclockwise rotation of 90° or 180° about its center.	 No	 Yes; the figure can be mapped onto itself by a clockwise or counterclockwise rotation of 72° or 144° about its center.