

# 7.5

## Glide Reflections and Compositions

- Goals**
- Identify glide reflections in a plane.
  - Represent transformations as compositions of simpler transformations.

### VOCABULARY

**Glide reflection** A glide reflection is a transformation in which every point  $P$  is mapped onto a point  $P''$  by the following two steps: (1) A translation maps  $P$  onto  $P'$ . (2) A reflection in a line  $k$  parallel to the direction of the translation maps  $P'$  onto  $P''$ .

**Composition** When two or more transformations are combined to produce a single transformation, the result is called a composition of the transformations.

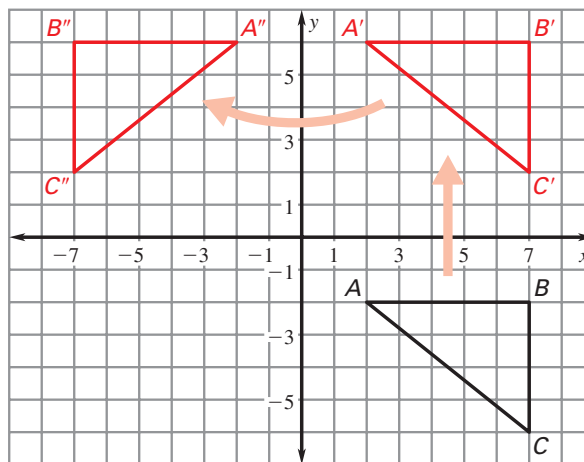
### Example 1 Finding the Image of a Glide Reflection

Sketch the image of  $\triangle ABC$  after a glide reflection.

**Translation:**  $(x, y) \rightarrow (x, y + 8)$

**Reflection:** in the  $y$ -axis

Translate  $\triangle ABC$  by moving it 8 units up to produce  $\triangle A'B'C'$ . The vertices of  $\triangle A'B'C'$  are  $A'(2, 6)$ ,  $B'(7, 6)$ , and  $C'(7, 2)$ . Next, reflect  $\triangle A'B'C'$  in the  $y$ -axis to produce  $\triangle A''B''C''$ . Its vertices are  $A''(-2, 6)$ ,  $B''(-7, 6)$ , and  $C''(-7, 2)$ .



- ✓ **Checkpoint** Write the coordinates of the image of  $P(4, -2)$  after the given glide reflection.

<p><b>1. Translation:</b> <math>(x, y) \rightarrow (x + 4, y)</math>  <b>Reflection:</b> in the <math>x</math>-axis  <math>(8, 2)</math></p>	<p><b>2. Translation:</b> <math>(x, y) \rightarrow (x, y - 6)</math>  <b>Reflection:</b> in the <math>y</math>-axis  <math>(-4, -8)</math></p>
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### THEOREM 7.6: COMPOSITION THEOREM

The composition of two (or more) isometries is an isometry.

#### Example 2 Finding the Image of a Composition

Sketch the image of  $\overline{MN}$  after a composition of the given rotation and reflection.

$$M(-4, 2), N(-2, 5)$$

**Rotation:**  $90^\circ$  clockwise about the origin

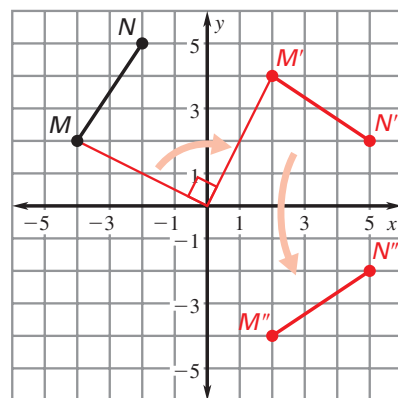
**Reflection:** in the  $x$ -axis

#### Solution

The graph of  $\overline{MN}$  is shown.

Rotate  $\overline{MN}$   $90^\circ$  about the origin to produce  $\overline{M'N'}$ . The endpoints of  $\overline{M'N'}$  are  $M'(2, 4)$  and  $N'(5, 2)$ .

Reflect  $\overline{M'N'}$  in the  $x$ -axis to produce  $\overline{M''N''}$ . The endpoints of  $\overline{M''N''}$  are  $M''(2, -4)$  and  $N''(5, -2)$ .

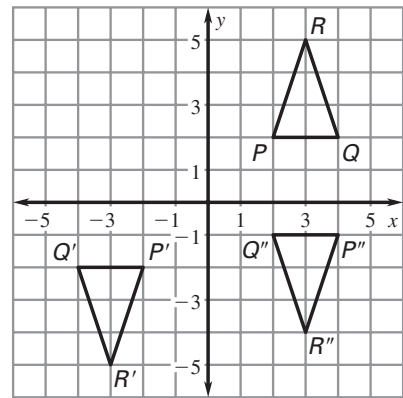


**Example 3** Describing a Composition

Describe the composition of transformations in the diagram.

**Solution**

Two transformations are shown. First,  $\triangle PQR$  is rotated  $180^\circ$  about the origin to produce  $\triangle P'Q'R'$ . Then  $\triangle P'Q'R'$  is translated using  $(x, y) \rightarrow (x + 6, y + 1)$  to produce  $\triangle P''Q''R''$ .



✔ **Checkpoint** Write the coordinates of the image of  $A(-3, 5)$  after a composition using the given transformations in the order they appear.

<p>3. Translation: <math>(x, y) \rightarrow (x + 9, y - 8)</math>                  Rotation: <math>90^\circ</math> counterclockwise about the origin</p> <p><b>(3, 6)</b></p>	<p>4. Rotation: <math>180^\circ</math> about the origin                  Reflection: in the y-axis</p> <p><b>(-3, -5)</b></p>
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Describe the composition of the transformations.

<p>5.</p> <p>Translation <math>(x, y) \rightarrow (x + 7, y)</math>, followed by a <math>180^\circ</math> rotation about the origin</p>	<p>6.</p> <p>Reflection in the x-axis, followed by a reflection in the line <math>x = 6</math></p>
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