

7.4

Translations and Vectors

- Goals**
- Identify and use translations in the plane.
 - Use vectors to describe translations.

VOCABULARY

Translation A translation is a transformation that maps every two points P and Q in the plane to points P' and Q' so that $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$.

Vector A vector is a quantity that has both direction and magnitude, and is represented by an arrow drawn between two points.

Initial point The initial point of a vector is the starting point of the vector.

Terminal point The terminal point of a vector is the ending point of the vector.

Component form The component form of a vector is the form that combines the horizontal and vertical components of the vector.

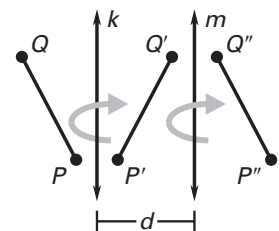
THEOREM 7.4: TRANSLATION THEOREM

A translation is an isometry.

THEOREM 7.5

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is a translation. If P'' is the image of P , then the following is true:

1. $\overleftrightarrow{PP'}$ is perpendicular to k and m .
2. $PP'' = \underline{2d}$ where d is the distance between k and m .



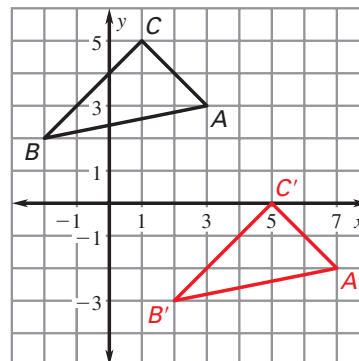
Example 1 *Translations in a Coordinate Plane*

Sketch the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x + 4, y - 5)$.

Graph $\triangle A'B'C'$ by shifting each point 4 units to the right and 5 units down.

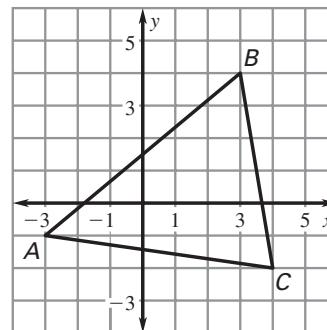
The coordinates of $\triangle ABC$ are listed below. Write the coordinates of $\triangle A'B'C'$.

$\triangle ABC$	$\triangle A'B'C'$
$A(3, 3)$	$A'(\underline{7}, \underline{-2})$
$B(-2, 2)$	$B'(\underline{2}, \underline{-3})$
$C(1, 5)$	$C'(\underline{5}, \underline{0})$

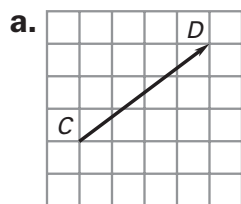


Checkpoint Write the coordinates of the vertices of the image after the given translation of $\triangle ABC$.

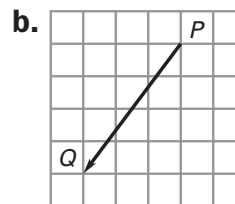
1. $(x, y) \rightarrow (x - 6, y + 3)$
 $A'(-9, 2), B'(-3, 7), C'(-2, 1)$

**Example 2** *Identifying Vector Components*

Name the vector and write its component form.



The vector is $\overrightarrow{CD} = \langle 4, 3 \rangle$.



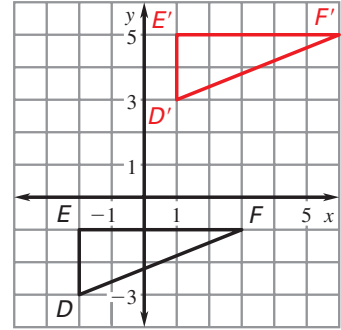
The vector is $\overrightarrow{PQ} = \langle -3, -4 \rangle$.

Example 3 Translation Using Vectors

The component form of \vec{JK} is $\langle 3, 6 \rangle$. Use \vec{JK} to translate $\triangle DEF$.

The component form of \vec{JK} is $\langle 3, 6 \rangle$, so the image vertices should be 3 units to the right and 6 units up from the preimage vertices. Graph and label the image vertices. Then use a straightedge to draw $\triangle D'E'F'$.

The vertices of $\triangle D'E'F'$ are $D'(\underline{1}, \underline{3})$, $E'(\underline{1}, \underline{5})$, and $F'(\underline{6}, \underline{5})$.



✔ **Checkpoint** Write the coordinates of the vertices of the image produced by translating $\triangle ABC$ using the given vector.

2. $\langle -4, 8 \rangle$

$A'(-6, 13), B'(-1, 8), C'(-3, 5)$

3. $\langle 0, -6 \rangle$

$A'(-2, -1), B'(3, -6), C'(1, -9)$

4. $\langle 7, 0 \rangle$

$A'(5, 5), B'(10, 0), C'(8, -3)$

