8.1 Ratio and Proportion

Goals • Find and simplify the ratio of two numbers.
• Use proportions to solve real-life problems.

VOCABULARY

Ratio of *a* to *b* If *a* and *b* are two quantities that are measured in the same units, then the ratio of *a* to *b* is $\frac{a}{b}$.

Proportion An equation that equates two ratios is a proportion.

Extremes In the proportion $\frac{a}{b} = \frac{c}{d}$, *a* and *d* are the extremes.

Means In the proportion $\frac{a}{b} = \frac{c}{d}$, *b* and *c* are the means.

Example 1 ***Simplifying Ratios***

Simplify the ratio.

a. $\frac{16 \text{ kg}}{800 \text{ g}}$ **b.** $\frac{7 \text{ ft}}{21 \text{ yd}}$

Solution

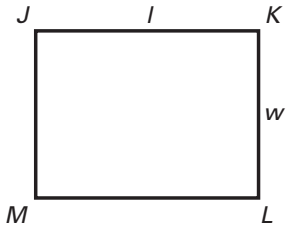
To simplify ratios with unlike units, convert to like units so that the units divide out. Then simplify the fraction, if possible.

a. $\frac{16 \text{ kg}}{800 \text{ g}} = \frac{16 \cdot \boxed{1000} \text{ g}}{800 \text{ g}} = \frac{\boxed{16{,}000}}{800} = \underline{20}$

b. $\frac{7 \text{ ft}}{21 \text{ yd}} = \underline{\frac{7 \text{ ft}}{21 \cdot 3 \text{ ft}}} = \underline{\frac{7}{63}} = \underline{\frac{1}{9}}$

Example 2 *Using Ratios*

The perimeter of rectangle *JKLM* is 56 centimeters. The ratio of *JK* : *KL* is 4 : 3. Find the length and width of the rectangle.



Solution

Because the ratio of *JK* : *KL* is 4 : 3, you can represent the length of *JK* as $4x$ and the width of *KL* as $3x$.

$2l + 2w = P$	**Formula for perimeter of rectangle**
$2(4x) + 2(3x) = 56$	**Substitute for *l*, *w*, and *P*.**
$8x + 6x = 56$	**Multiply.**
$14x = 56$	**Combine like terms.**
$x = 4$	**Divide each side by 14.**

Answer So, *JKLM* has a length of 16 centimeters and a width of 12 centimeters.

✔ ***Checkpoint*** **Simplify the ratio.**

1. $\frac{5 \text{ km}}{200 \text{ m}}$ 25 : 1	**2.** $\frac{40 \text{ oz}}{5 \text{ lb}}$ 1 : 2

PROPERTIES OF PROPORTIONS

1. Cross Product Property The product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

2. Reciprocal Property If two ratios are equal, then their reciprocals are also equal.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

Example 3 *Solving Proportions*

Solve the proportion $\frac{t+4}{8} = \frac{t}{3}$.

$\frac{t+4}{8} = \frac{t}{3}$ **Write original proportion.**

$3(t+4) = 8t$ **Cross product property**

$3t + 12 = 8t$ **Distributive property**

$\frac{12}{5} = t$ **Simplify.**

Example 4 *Solving a Proportion*

A scale model of a car is 10 inches long and 5 inches tall. The actual car is 60 inches tall. What is the length of the actual car?

Verbal Model $\frac{\text{Length of car}}{\text{Length of model}} = \frac{\text{Height of car}}{\text{Height of model}}$

Labels Length of car $= x$ Height of car $=$ 60 (in.)

Length of model $=$ 10 Height of model $=$ 5 (in.)

Reasoning $\frac{x}{10} = \frac{60}{5}$ **Substitute.**

$x = 10\left(\frac{60}{5}\right)$ **Multiply each side by** 10.

$x =$ 120 **Simplify.**

Answer So, the actual car is 120 inches, or 10 feet long.

✔ ***Checkpoint*** **Solve the proportion.**

3. $\frac{7}{2} = \frac{21}{r}$ 6	**4.** $\frac{6}{3x-12} = \frac{4}{x}$ 8