

8.2

Problem Solving in Geometry with Proportions

- Goals**
- Use properties of proportions.
 - Use proportions to solve real-life problems.

VOCABULARY

Geometric mean The geometric mean of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$, or $x = \sqrt{a \cdot b}$.

ADDITIONAL PROPERTIES OF PROPORTIONS

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Example 1 Using Properties of Proportions

Tell whether the statement is true.

If $\frac{v}{5} = \frac{w}{9}$, then $\frac{v+5}{5} = \frac{w+5}{9}$.

Solution

$\frac{v}{5} = \frac{w}{9}$ Given

$\frac{v+5}{5} = \frac{w+9}{9}$ If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Because $\frac{w+9}{9} \neq \frac{w+5}{9}$, the conclusions are

not equivalent.

Answer The statement is false.

Example 2 Using Properties of Proportions

In the diagram $\frac{HJ}{JL} = \frac{GK}{KL}$. Find the length of \overline{GK} .

Solution

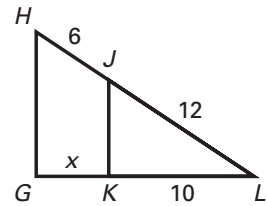
$$\frac{HJ}{JL} = \frac{GK}{KL} \quad \text{Given.}$$

$$\frac{6}{12} = \frac{x}{10} \quad \text{Substitute.}$$

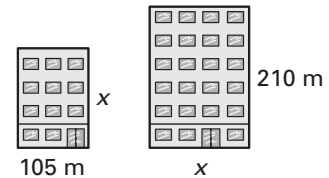
$$12x = 60 \quad \text{Cross product property}$$

$$x = 5 \quad \text{Divide each side by } 12.$$

Answer So, the length of \overline{GK} is 5.

**Example 3** Using a Geometric Mean

The two buildings shown have the same width-to-length ratio. The distance labeled x is the geometric mean of 105 m and 210 m. Find the value of x .

**Solution**

$$\frac{105}{x} = \frac{x}{210} \quad \text{Write proportion.}$$

$$x^2 = 105 \cdot 210 \quad \text{Cross product property}$$

$$x = \sqrt{105 \cdot 210} \quad \text{Simplify.}$$

$$x = \sqrt{105 \cdot 105 \cdot 2} \quad \text{Factor.}$$

$$x = 105\sqrt{2} \quad \text{Simplify.}$$

Answer The geometric mean of 105 and 210 is $105\sqrt{2}$, or about 148. So, the distance labeled x in the diagram is about 148 m.

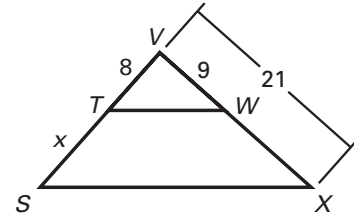
✓ **Checkpoint** Complete the following exercises.

1. Tell whether the statement is true: If $\frac{m}{7} = \frac{n}{11}$, then $\frac{m}{n} = \frac{7}{11}$.

True

2. In the diagram, $\frac{ST}{TV} = \frac{WX}{VW}$. Find ST .

$10\frac{2}{3}$

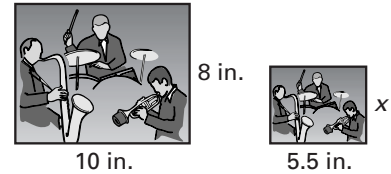


3. Find the geometric mean of 54 and 192.

$72\sqrt{2}$

Example 4 Solving a Proportion

You have a 10-by-8 inch photo of the school band that must be reduced to a length of 5.5 inches for the school yearbook. What is the width of the reduced photo?



Solution

Verbal Model $\frac{\text{Length of original}}{\text{Length of reduced}} = \frac{\text{Width of original}}{\text{Width of reduced}}$

Labels
 Length of original photo = 10 (inches)
 Length of reduced photo = 5.5 (inches)
 Width of original photo = 8 (inches)
 Width of reduced photo = x (inches)

Reasoning $\frac{10}{5.5} = \frac{8}{x}$ **Substitute.**
 $10x = 8(5.5)$ **Cross product property**
 $x = 4.4$ **Use a calculator.**

Answer So, the reduced photo has a width of 4.4 inches.