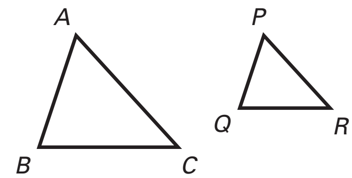
8.5 Proving Triangles are Similar

Goals
- Use similarity theorems to prove two triangles are similar.
- Use similar triangles to solve real-life problems.

THEOREM 8.2: SIDE-SIDE-SIDE (SSS) SIMILARITY THEOREM

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

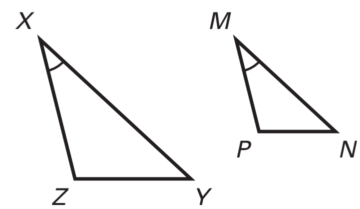


If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$,

then $\triangle$ ABC $\sim \triangle$ PQR.

THEOREM 8.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

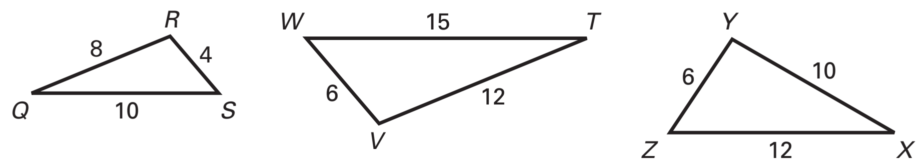


If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$,

then $\triangle$ XYZ $\sim \triangle$ MNP.

Example 1 *Using the SSS Similarity Theorem*

Which of the following three triangles are similar?



To decide which, if any, of the triangles are similar, you need to consider the ratios of the lengths of corresponding sides.

Ratios of Side Lengths of $\triangle QRS$ and $\triangle TVW$

Shortest sides	Longest sides	Remaining sides
$\frac{RS}{VW} = \frac{4}{6} = \frac{2}{3}$,	$\frac{QS}{TW} = \frac{10}{15} = \frac{2}{3}$,	$\frac{QR}{TV} = \frac{8}{12} = \frac{2}{3}$

Answer Because the ratios are equal, $\triangle QRS \sim \triangle TVW$.

Ratios of Side Lengths of $\triangle QRS$ and $\triangle XYZ$

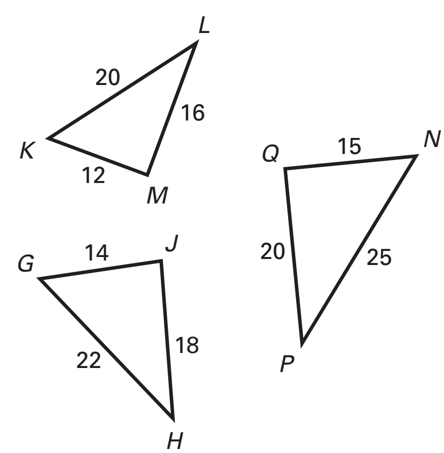
Shortest sides	Longest sides	Remaining sides
$\frac{RS}{YZ} = \frac{4}{6} = \frac{2}{3}$,	$\frac{QS}{XZ} = \frac{10}{12} = \frac{5}{6}$,	$\frac{QR}{XY} = \frac{8}{10} = \frac{4}{5}$

Answer Because the ratios are not equal, $\triangle QRS$ and $\triangle XYZ$ are not similar.

Checkpoint **Complete the following exercise.**

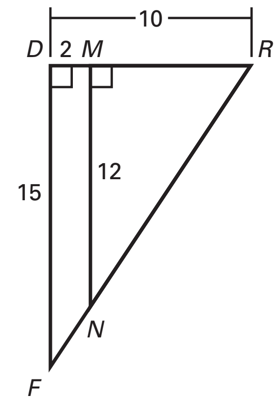
1. Which of the three triangles are similar?

$\triangle KLM$ and $\triangle NPQ$



Example 2 ***Using the SAS Similarity Theorem***

Use the given lengths to prove that $\triangle DFR \sim \triangle MNR$.



Solution

Given: $DF = 15$, $MN = 12$
$DM = 2$, $DR = 10$

Prove: $\triangle DFR \sim \triangle MNR$

Paragraph Proof Use the SAS Similarity Theorem. Begin by finding the ratios of the lengths of the corresponding sides.

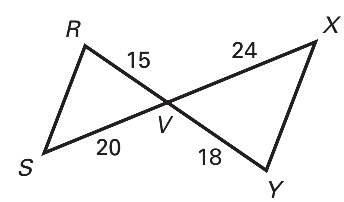
$$\frac{DF}{MN} = \frac{15}{12} = \frac{5}{4}$$

$$\frac{DR}{MR} = \frac{10}{10 - 2} = \frac{10}{8} = \frac{5}{4}$$

So, the lengths of sides $\overline{DF}$ and $\overline{DR}$ are proportional to the lengths of the corresponding sides of $\triangle MNR$. Because $\angle FDR$ and $\angle NMR$ are right angles, use the SAS Similarity Theorem to conclude that $\triangle DFR \sim \triangle MNR$.

✔ ***Checkpoint*** **Complete the following exercise.**

2. Describe how to prove that $\triangle RSV$ is similar to $\triangle YXV$.



Show that corresponding sides are proportional. Then use the SAS Similarity Theorem with the vertical angles $\angle RVS$ and $\angle YVX$.