

# 8.6

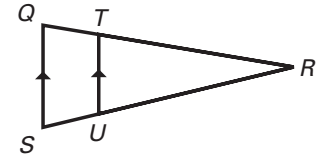
## Proportions and Similar Triangles

- Goals**
- Use proportionality theorems to calculate segment lengths.
  - Solve real-life problems.

### THEOREM 8.4: TRIANGLE PROPORTIONALITY THEOREM

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

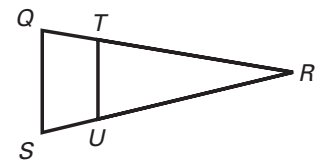
$$\text{If } \overline{TU} \parallel \overline{QS}, \text{ then } \frac{RT}{TQ} = \frac{RU}{US}.$$



### THEOREM 8.5: CONVERSE OF THE TRIANGLE PROPORTIONALITY THEOREM

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

$$\text{If } \frac{RT}{TQ} = \frac{RU}{US}, \text{ then } \overline{TU} \parallel \overline{QS}.$$



### Example 1 Finding the Length of a Segment

What is the length of  $\overline{NR}$ ?

#### Solution

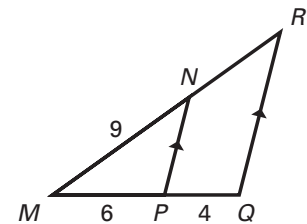
$$\frac{PQ}{MP} = \frac{NR}{MN} \quad \text{Theorem } \underline{8.4}$$

$$\frac{\boxed{4}}{\boxed{6}} = \frac{NR}{\boxed{9}} \quad \text{Substitute.}$$

$$\frac{4(9)}{6} = NR \quad \text{Multiply each side by } \underline{9}.$$

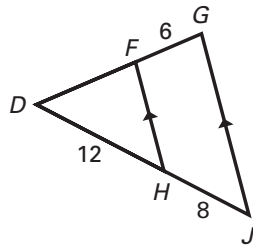
$$\underline{6} = NR \quad \text{Simplify.}$$

**Answer** So, the length of  $\overline{NR}$  is 6.



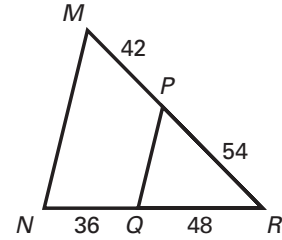
✔ **Checkpoint** Complete the following exercises.

1. Find the length of  $\overline{DF}$ .



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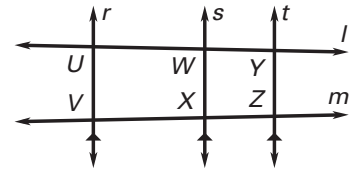
2. Given the diagram, determine whether  $\overline{MN}$  is parallel to  $\overline{PQ}$ .



no

**THEOREM 8.6**

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

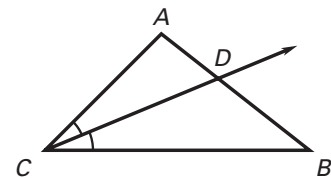


If  $r \parallel s$  and  $s \parallel t$ , and  $l$  and  $m$

intersect  $r$ ,  $s$ , and  $t$ , then  $\frac{UW}{WY} = \frac{VX}{XZ}$ .

**THEOREM 8.7**

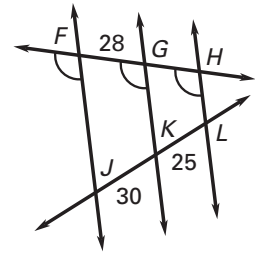
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



If  $\overrightarrow{CD}$  bisects  $\angle ACB$ , then  $\frac{AD}{DB} = \frac{CA}{CB}$ .

**Example 2** Using Proportionality Theorems

What is the length of  $\overline{GH}$ ?

**Solution**

Because corresponding angles are congruent, the lines are parallel and you can use Theorem 8.6.

$$\frac{FG}{GH} = \frac{JK}{KL}$$

Parallel lines divide transversals proportionally.

$$\frac{28}{GH} = \frac{30}{25}$$

Substitute.

$$28 \cdot 25 = 30 \cdot GH$$

Cross product property.

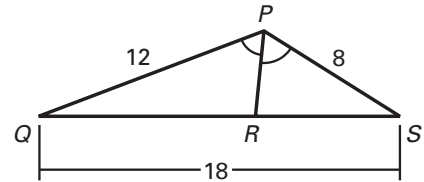
$$\frac{70}{3} = GH$$

Divide each side by 30 and simplify.

Answer So, the length of  $\overline{GH}$  is  $\frac{70}{3}$ , or  $23\frac{1}{3}$ .

**Example 3** Using Proportionality Theorems

In the diagram,  $\angle QPR \cong \angle RPS$ . Use the given side lengths to find the length of  $\overline{QR}$ .

**Solution**

Because  $\overline{PR}$  is an angle bisector of  $\angle QPS$ , you can apply Theorem 8.7.

Let  $x = QR$ . Then,  $RS = 18 - x$ .

$$\frac{PS}{QP} = \frac{RS}{QR}$$

Apply Theorem 8.7.

$$\frac{8}{12} = \frac{18 - x}{x}$$

Substitute.

$$8 \cdot x = 12(18 - x)$$

Cross product property

$$8x = 216 - 12x$$

Distributive property

$$20x = 216$$

Add  $12x$  to each side.

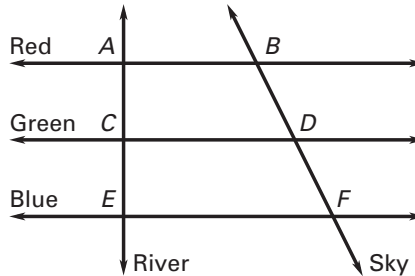
$$x = 10.8$$

Divide each side by 20.

Answer So, the length of  $\overline{QR}$  is 10.8 units.

**Example 4** Finding the Length of a Segment

**Hiking Trails** A state park has five trails, as shown in the diagram. The horizontal Red, Green, and Blue trails are evenly spaced. The River and Sky trails intersect the Red, Green and Blue trails. Explain why the distance between the Red and Green trails on the Sky trail is the same as the distance between the Green and Blue trails on the Sky trail.

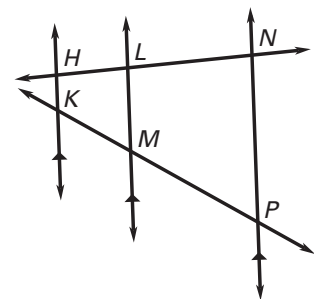


**Solution**

Because the Red, Green, and Blue trails are each horizontal, you know that they are parallel to each other. Using Theorem 8.6, you can conclude that  $\frac{AC}{CE} = \frac{BD}{DF}$ . Because the trails are evenly spaced, you know that  $AC = CE$ . So, you can conclude that  $BD = DF$ , which means that the distance between the Red and Green trails and the Green and Blue trails on the Sky trail have the same lengths.

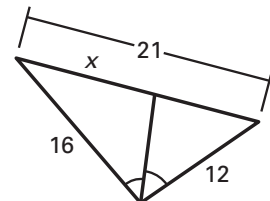
✔ **Checkpoint** Complete the following exercises.

3. In the diagram,  $\overline{HK} \parallel \overline{LM} \parallel \overline{NP}$ ,  $HL = 2.4$ ,  $KM = 2.8$ , and  $MP = 4.9$ . Find the length of  $\overline{LN}$ .



4.2

4. Find the value of  $x$ .



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