8.7 Dilations

Goals
- Identify dilations.
- Use properties of dilations in real-life.

VOCABULARY

Dilation A dilation with center C and scale factor k is a transformation that maps every point P in the plane to a point P' so that the following properties are true.
(1) If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$, and $k \neq 1$. (2) If P is the center point C, then $P = P'$.

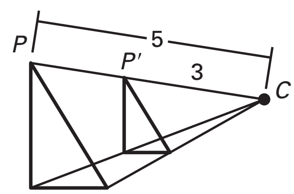
Enlargement A dilation with $k > 1$

Reduction A dilation with $0 < k < 1$

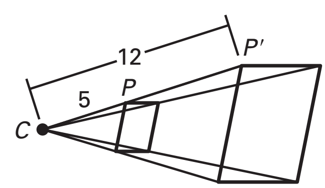
Example 1 *Identifying Dilations*

Identify the dilation and find its scale factor.

a.



b.



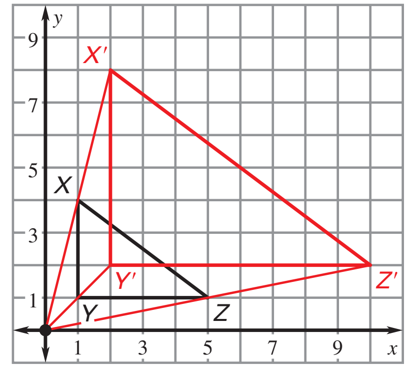
Solution

a. Because $\frac{CP'}{CP} = \frac{3}{5}$, the scale factor is $k = \frac{3}{5}$. This is a reduction.

b. Because $\frac{CP'}{CP} = \frac{12}{5}$, the scale factor is $k = \frac{12}{5}$. This is an enlargement.

Example 2 *Dilation in a Coordinate Plane*

Draw a dilation of $\triangle XYZ$. Use the origin as the center and use a scale factor of 2. Find the perimeter of the preimage and the perimeter of the image.



Solution

Because the center of the dilation is the origin, you can find the image of each vertex by multiplying its coordinates by the scale factor.

$X(1, 4) \to X'(2, 8)$

$Y(1, 1) \to Y'(2, 2)$

$Z(5, 1) \to Z'(10, 2)$

To find the perimeters of the preimage and image, you need to find XZ and $X'Z'$.

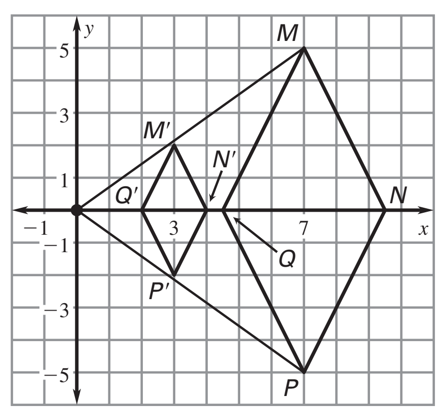
$$XZ = \sqrt{(1 - 5)^2 + (4 - 1)^2} = \sqrt{(-4)^2 + 3^2}$$
$$= \sqrt{16 + 9} = \sqrt{25} = 5$$

Perimeter of $\triangle XYZ = 3 + 4 + 5 = 12$

$$X'Z' = \sqrt{(2 - 10)^2 + (8 - 2)^2} = \sqrt{(-8)^2 + 6^2}$$
$$= \sqrt{64 + 36} = \sqrt{100} = 10$$

Perimeter of $\triangle X'Y'Z' = 6 + 8 + 10 = 24$

Checkpoint Use the dilation shown.



1. Is the dilation shown a reduction or an enlargement?

   Reduction

2. What is the scale factor?

   2.5

3. What are the coordinates of the vertices of the image?

   $M'(3, 2)$, $N'(4, 0)$, $P'(3, -2)$, $Q'(2, 0)$