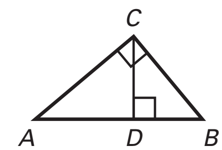
9.1 Similar Right Triangles

Goals
- Solve problems involving similar right triangles formed by the altitude drawn to the hypotenuse of a right triangle.
- Use a geometric mean to solve problems.

THEOREM 9.1

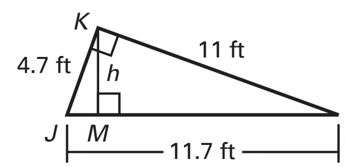
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

Example 1 ***Finding the Height of a Ramp***

Ramp Height A ramp has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the ramp.



Solution

By Theorem 9.1, $\triangle JKL \sim \triangle KML$.

Use similar triangles to write a proportion.

$\frac{KM}{JK} = \frac{KL}{JL}$ **Corresponding side lengths are in proportion.**

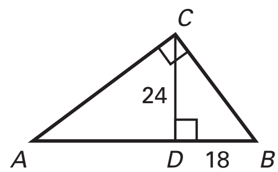
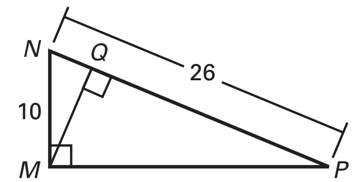
$\frac{h}{4.7} = \frac{11}{11.7}$ **Substitute.**

$11.7h = 11(4.7)$ **Cross product property**

$h \approx 4.4$ **Solve for h.**

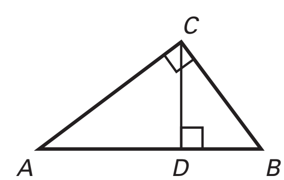
Answer The height of the ramp is about 4.4 feet.

Checkpoint **Write similarity statements for the three triangles in the diagram. Then find the given length. Round decimals to the nearest tenth, if necessary.**

1. Find AD.	**2.** Find NQ.
 *Sample answer:* $\triangle ABC \sim \triangle CBD \sim \triangle ACD$; 32	 *Sample answer:* $\triangle MNP \sim \triangle QNM \sim \triangle QMP$; 3.8

GEOMETRIC MEAN THEOREMS

THEOREM 9.2



In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 9.3

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

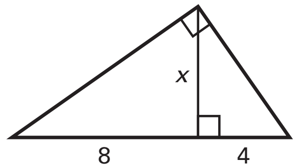
$$\frac{AB}{CB} = \frac{CB}{DB}$$

$$\frac{AB}{AC} = \frac{AC}{AD}$$

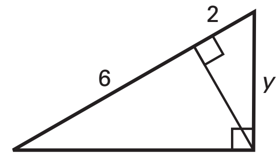
Example 2 *Using a Geometric Mean*

Find the value of each variable.

a.



b.



Solution

a. Apply Theorem 9.2.

$$\frac{\boxed{8}}{x} = \frac{x}{4}$$

$$32 = x^2$$

$$\sqrt{32} = x$$

$$4\sqrt{2} = x$$

b. Apply Theorem 9.3.

$$\frac{6 + 2}{y} = \frac{y}{\boxed{2}}$$

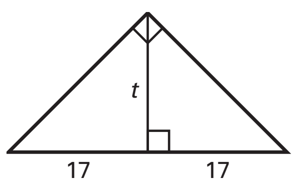
$$\frac{8}{y} = \frac{y}{\boxed{2}}$$

$$16 = y^2$$

$$4 = y$$

Checkpoint Find the value of the variable.

3.



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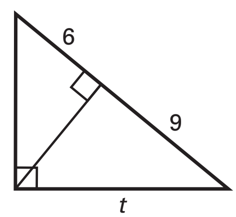
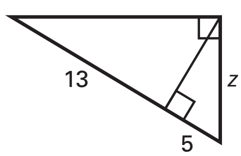
4.



$3\sqrt{15}$

5.



$3\sqrt{10}$